Behaviour of Reinforced Concrete Frames under Non-proportional Loadings

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Research Report No. R 146
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ISBN 0 86396 416 8

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ABSTRACT: This report gives details of a study of the overload behaviour and strength of concrete framed structures when subjected to non-proportional loadings. A special analytic procedure was developed to simulate the overload behaviour and collapse of a concrete frame when subjected to vertical and horizontal loads which are applied in sequence.

Comparisons were made of the effects of applying loads sequentially and simultaneously to a variety of structural systems. It is concluded that load capacity for sequentially applied loads can be predicted with good accuracy by using an equivalent proportional load system.

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1. INTRODUCTION

Concrete structures are designed to resist various combinations of loads, which act both vertically and horizontally. To simplify the design calculations, it is always assumed that the loads comprising any one combination are applied simultaneously and proportionally, and the load effects are usually evaluated by means of linear elastic frame analysis.

In reality, the loads always occur in sequence. For example, the vertical loads due to self weight and dead and live loads preceding the horizontal loads due to wind or earthquake. Furthermore, concrete structures behave in a highly non-linear manner, even under working load conditions. Although non-linear analysis packages are gradually becoming available commercially, they usually predict structural behaviour under conditions of a progressively increasing, but proportional, system of loads.

This paper reports on an investigation of concrete structural behaviour under non-proportional loading. The aim of the study was to evaluate the effect of the sequence of loading on the load capacity of concrete frames. The situation to be investigated is illustrated in Fig. 1.

In Fig. 1a, a simple frame is considered with a vertical load system V and a horizontal load system H. Three load sequences are considered. In the first sequence, Fig. 1b, the vertical load is increased to a value represented by V_{1max} and the horizontal load is then progressively increased until collapse occurs at the value H_{1ult}. In the second sequence, Fig. 1c, the horizontal load is first increased to a maximum value of H_{2max}, which is chosen to be equal to H_{1ult}, and the vertical load is then increased until failure occurs at V_{2ult}. In the third case, Fig. 1d, the vertical and horizontal loads are applied simultaneously. They are proportionally increased in the ratio $V_{1max}:H_{1ult}$ until failure occurs at loads of V_{3ult} and H_{3ult}. Clearly, the load combinations at collapse are not necessarily the same in these three cases. Unfortunately, very little research has been undertaken on the effects of load sequences and non-proportional loading (Kenyon and Warner, 1993), and the purpose of the present study was to obtain the quantitative estimates of the effect of load sequence on load carrying capacity.

2. METHOD OF ANALYSIS

In order to undertake the investigation, a computational procedure was developed to simulate the sequential application of the loads. Only two load systems (Fig. 1a) were considered in the present study, in order to limit the computations, however, the method can easily be extended to treat any number of load systems. The first load system is progressively increased to a pre-determined value and maintained. The second load system is then progressively increased on the loaded structure until a condition of collapse is achieved.

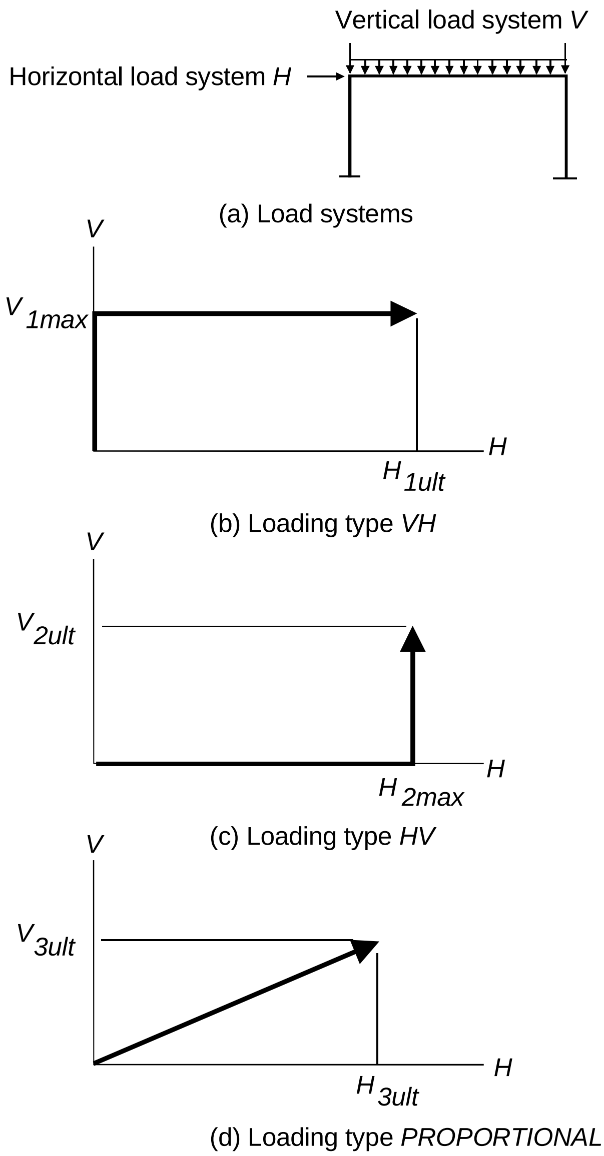

Figure 1: Diagrams showing the various loading paths

The simulation program is part of an ongoing study of the non-linear behaviour of concrete structures (Warner, 1975; Warner and Yeo, 1984a, 1984b; Wong, Yeo and Warner, 1988, 1990; Wong and Warner, 1988, 1990). Non-linear analysis can be undertaken using deformation control, load control, or a combined load-deformation control. Load-control methods tend to have convergence problems when used to determine load capacity, although methods have been devised to deal with peak loads and post failure decreasing loads. Deformation control procedures have been developed which use curvature control in a local region (Wong, Yeo and Warner, 1988), and deflection control at a suitable point (Kawano and Warner, 1995). An early application of the combined load-deformation control is the approach of Crisfield (1983), which is very effective although computationally intensive in dealing with peak load behaviour. While the deformation control and load-deformation control methods are both suitable for determining the behaviour of the structure at collapse under proportional loading, the curvature control method was found in the present study to provide a simple and efficient basis for the analysis of non-proportional loading. In a previous preliminary study, Kenyon and Warner (1993) used load control for the first load system and the first part of the second load application and then changed over to defomation control for the second part of the second load application to failure. In the present study, the use of curvature control throughout has greatly simplified the computational procedure.

The computational procedure for using curvature control for proportional analysis is briefly describe in Appendix A. A more detailed description of the procedure can be found in an earlier paper (Wong, Yeo and Warner, 1988). The extension of this procedure to non-proportional analysis will now be explained.

The two load systems are represented by two unit load patterns which are progressively increased by means of scaling factors.

In Fig. 1a, the rectangular portal frame is subjected to a vertical load system consisting of a uniformly distributed load of w kN/m and two point loads of αwL kN which act on each column. To represent various levels of loading, an initial value of w (eg 1.0 kN/m) is taken, and a progressively increasing scaling factor λ_1 is introduced. For the horizontal load system, a unit load H (eg 1.0 kN) is scaled using λ_2.

Load pattern 1 is applied in several steps by increasing the load factor λ_1 in steps using curvature control until it reaches λ_{1max}. From this stage on, λ_1 is kept at this value while curvature control is used to increase λ_2 until collapse occurs at λ_{2ult}.

As in the proportional analysis described in Appendix A, the frame is modelled by dividing it into numerous segments and each of these segments is assumed to have a uniform flexural stiffness equal to that of the most critically stressed section within it. The section behaviour is modelled by dividing it into numerous concrete and steel layers. The concrete layers are assumed to have a non-linear stress-strain relation, and the steel layers are assumed to have a linearly elastic-plastic stress-strain relation. These stress-strain relations allow for unloading paths due to strain reversal. Structural members such as beams and columns are assumed to have equal size segments. The number of segments chosen to represent each member gives a length to depth ratio of approximately unity.

An incremental-iterative curvature control solution strategy is used. The full range behaviour is obtained by subjecting a nominated key segment to increasing curvatures. This key segment is selected at the start of the analysis. A basis for this selection is that it has monotonically increasing curvatures as the loading process progresses, initially under the influence of load pattern 1 and subsequently under the additional influence of load pattern 2. At the start of a typical curvature-increment step, the targeted curvature for the key segment is calculated by adding a preset curvature increment to the targeted curvature of the preceding step. The load factors λ_1 and λ_2, consistent with this targeted curvature in the key segment, are then determined by carrying out several iterative cycles of calculations until convergence is achieved.

Computational steps required for a typical iterative cycle within an incremental step which has been assigned a targeted curvature K_{step} are:

1. The frame with the latest full-load stiffnesses EI_i in all the segments is subjected to the unit load pattern 1. A second order elastic analysis routine is used to determine unit curvatures k_{11}, k_{21}, ..., k_{key1}, k_{i1}, ..., k_{nseg1} in all the segments. Similarly, the frame with the latest full-load stiffnesses EI_i in all the segments is subjected to the unit load pattern 2. A second order elastic analysis routine is used also to determine unit curvatures k_{12}, k_{22}, ..., k_{key2}, k_{i2}, ..., k_{nseg2} in all the segments. If

this is the first cycle, then these stiffnesses are set to those calculated at the end of the previous step, else these stiffnesses are set to those calculated at the end of the previous cycle.

2. The load factor λ_1 to give the key segment a total curvature (the term total curvature means curvature under full loading) of K_{step} is then estimated :

$$\lambda_1 = \frac{K_{step}}{|k_{key1}|} \tag{1}$$

where k_{key1} is the curvature of key segment under unit load pattern 1.

3. If λ_1 is less than or equal to λ_{1max}, λ_1 is taken to be the value calculated in step 2 above and λ_2 is set to zero. Proceed to Step 5.

4. If λ_1 exceeds λ_{1max} the following steps are required:

   - Estimate the full load curvature of the key segment under load pattern 1 K_{key1} which will give a λ_1 value equal to λ_{1max} :

$$K_{key1} = \lambda_{1\max} \times |k_{key1}| \tag{2}$$

   - Load factor λ_1 is then set to λ_{1max} :

$$\lambda_1 = \lambda_{1\max} \tag{3}$$

   - The required total curvature of the key segment for the frame subjected to load pattern 2 to give a total curvature of K_{step} under the combined application of the two load patterns is calculated:

$$K_{key2} = K_{step} - K_{key1} \tag{4}$$

- The load factor λ_2 to give the key segment a total curvature of K_{key2} is then calculated:

$$\lambda_2 = \frac{K_{key2}}{\left|k_{key2}\right|} \tag{5}$$

where k_{key2} is the curvature of key segment under unit load pattern 2.

5. After having calculated both λ_1 and λ_2, they are used to scale the corresponding unit curvatures of the segments to give corresponding total curvatures. For a typical segment i, the principle of superposition is then used to calculate the total curvature K_i:

$$K_i = (\lambda_1 \times k_{i1}) + (\lambda_2 \times k_{i2}) \tag{6}$$

where k_{i1} is the curvature of the ith segment under unit load pattern 1 and k_{i2} is the curvature of the ith segment under unit load pattern 2. Note that the simultaneous application of λ_1 and λ_2 in the above equation will give a curvature of K_{step} in the key segment.

6. For each segment i with curvatures K_i , a section analysis routine which takes account of the material stress-strain relations is used to calculate the corresponding bending moment, M_i. The stiffnesses of all the segments are updated:

$$EI_i = \frac{M_i}{K_i} \tag{7}$$

7. Convergence of the EI_i values is checked at this stage. If convergence is achieved, a solution has been obtained for the present curvature step. The targeted curvature is then set to that of the next step. Proceed to step (1) to start a new set of iterative cycles. If convergence is not achieved then proceed to step (1) maintaining the present targeted curvature for more iterative cycles.

The solution procedure caters for both geometric and material nonlinearities. The geometrical nonlinearity is taken into consideration when the second order elastic analysis is carried out, and the material non-linearity is taken into consideration when the section analysis is used to update the stiffnesses of the segments.

3. FREE-STANDING COLUMN

Initially a 3.0m, uniform, free-standing, 300mm x 300mm square column shown in Fig. 2 was analysed. The following properties were assumed: f'_c = 32 MPa, f_{sy} = 400 MPa with mean values: f_{cm} = 37.5 MPa and f_{sm} = 460 MPa. The total amount of reinforcement is 1800 mm^2, with concrete cover to centroid of steel reinforcement of 40 mm. The peak stress used for the stress-strain relation of in-situ concrete is assumed to be 0.85 f_{cm}. This takes into account the difference between the strength of the concrete in the structure and the cylinder strength obtained from tests.

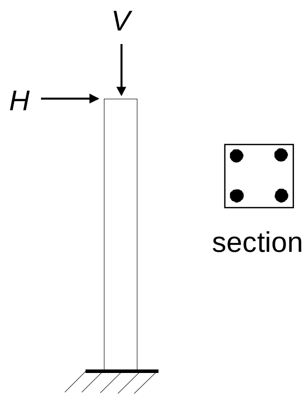

Figure 2: Free-standing Column

V is the vertical, downward point load acting at the top of the column and *H* is the horizontal point load acting at the top of the column as shown in Fig. 2. Three loading paths were considered. For the first, type *VH*, curvature control was used to increase the vertical load *V* until it reached a predetermined value V_{1max}. This was followed by curvature control to increase the horizontal load *H* until its peak value H_{1ult} was traversed. After having obtained H_{1ult} for the predetermined V_{1max}, another loading path, type *HV* in Fig. 1(c), was simulated. This caused an increment of *H* to H_{2max}, where H_{2max} is set to H_{1ult} obtained earlier, followed by increments of *V* until its peak value V_{2ult} was traversed. The final loading path *proportional* in Fig. 1(a), is with increments of *V* and *H* (the ratio of *V* to *H* maintained at V_{1max}:H_{1ult}) until their peak values H_{3ult} and V_{3ult} were traversed. This loading path will be referred to as the equivalent proportional load for the non-proportional loading type *VH* in this reports.

A strength interaction diagram based on values of (V_{1max}, H_{1ult}) for the non-proportional type *VH* is shown in Fig. 3 as a continuous line. This strength interaction line is plotted from results obtained using non-linear analyses and therefore includes the effect of material and geometric non-linearities. For comparison purposes, values of (V_{2ult}, H_{2max}) for the non-proportional type *HV* and (V_{3ult} , H_{3ult}) for the loading type *proportional* are also plotted on the same diagram.

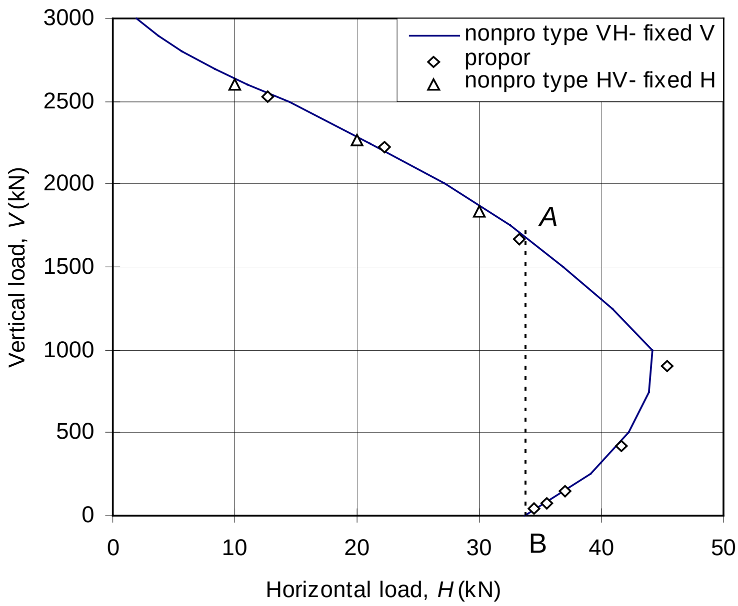

Figure 3: Interaction diagram for free-standing column

Fig. 3 shows that in the case where *H* is caused to increase first, it cannot be increased beyond a value indicated in the figure as point *B*. Therefore this loading type cannot be applied to give loading states represented by line *AB*. However, these states are attainable to both loading type *VH* and loading type *proportional*. This behaviour is consistent with previous observation on the stiffening effect of thrusts on concrete section behaviour (Ehsani and Alameddine, 1987).

In those cases where solutions are attainable, the peak loads predicted for the frames subjected to non-proportional loads (types *HV* and *VH)* agree well with those predicted using their corresponding equivalent proportional loads.

4. PORTAL FRAMES

The rectangular portal frame shown in Fig. 4 was first subjected to non-proportional load of type *VH*. The properties of the concrete and reinforcing steel are the same as those used for the column analysed earlier. The vertical pattern load was applied by increasing α until it reached a predetermined value α_{1max}. When α reached this value, the uniformly distributed load on the beam simultaneously reached a value of $0.5w_{beam}$, where w_{beam} is the failure load of the frame with only the vertical uniformly distributed load *w* acting along the beam. Limiting the maximum value of α to this value ensured that the frame did not failed prematurely by forming a local beam failure mechanism. Curvature control was used to increase the horizontal load until it reached its peak value of H_{1ult}. After having determined H_{1ult} for a given α_{1max}, the frame was then subjected to an equivalent proportional loading (ie with the H:α ratio maintained at H_{1ult}:α_{1max}) until failure occurred at (H_{3ult}:α_{3ult}). The frame was also subjected to non-proportional loading of type *HV*. For this loading type, *H* was increased until it reached a value of H_{2max} (where H_{2max} is set equal to H_{1ult}); it was then maintained constant while α was increased until it reached its peak value α_{2ult}. Results for the three loading types for the 4m tall frame are presented in the form of a strength interaction diagram shown on Fig. 5. Note that this interactive diagram also includes the effects of both geometric and material non-linearities.

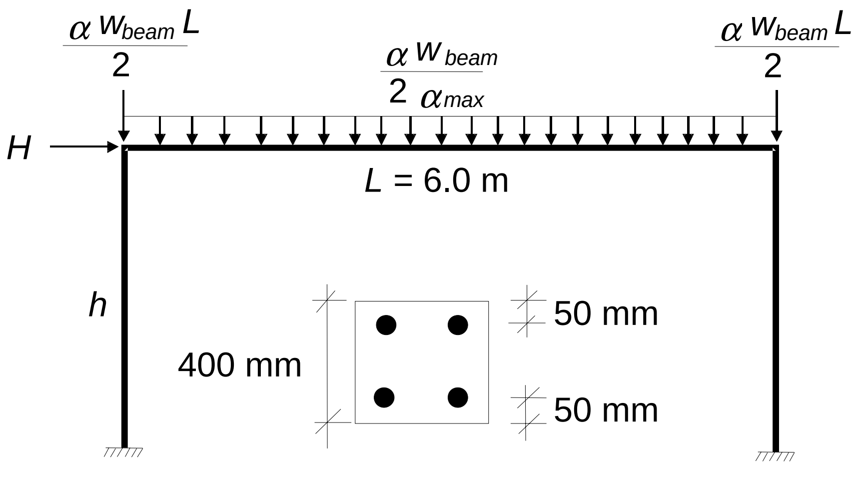

Figure 4: Portal Frame

The results show that, generally, equivalent proportional loading gives a good estimate of the peak loads for frames subjected to non-proportional loading. Similar to the observation earlier for the free-standing column, states represented by the portion of the interaction diagram to the right of the horizontal load *H* of 223 kN are not attainable using the loading type *HV*.

Horizontal load versus sway deflection curves for type *VH* corresponding to α_{max} of 10.0 are shown in Fig. 6. The sway deflection is taken as the horizontal deflection at the top of the left column of the portal frame. Results obtained for the equivalent proportional load, type *proportional*, are also plotted in the same figure. The results show that while the peak loads for both loading types are almost the same, type *VH* has a smaller sway stiffness over a large portion of the loading curves.

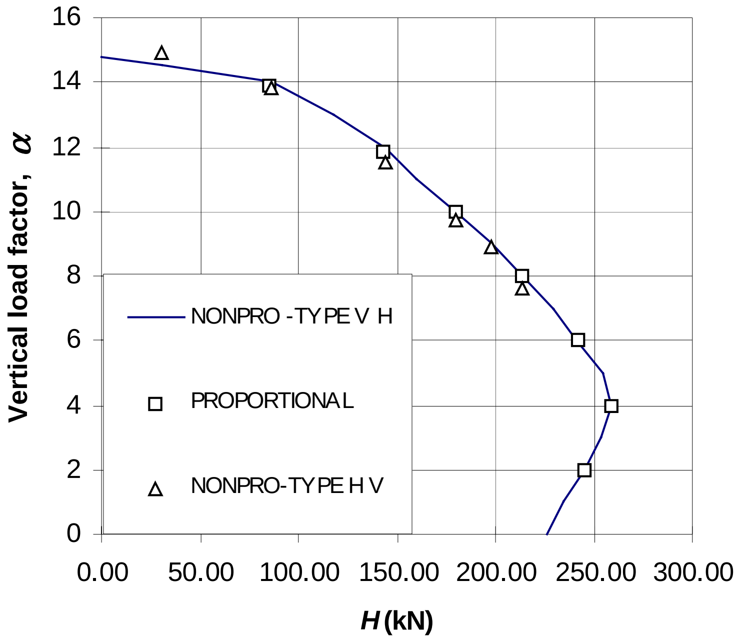

Figure 5: Strength interaction diagram for 4m tall portal frame

Curves of vertical load factor α versus sway deflection are shown in Fig. 7 for the non-proportional load type *HV* and the proportional type *proportional*. The sway deflection is also taken as the horizontal deflection at the top of the left column of the portal frame. The results show that for type *HV*, the lateral stiffness of the frame increases initially on the application of the vertical load. This behaviour is consistent with previous observations (Ehsani and Alameddine, 1987) of the stiffening effect of thrust on moment curvature relations of concrete sections.

The use of an equivalent proportional load for loading type *HV* underestimates the sway deflection of this frame over most of its loading history. At half the peak load level of α, the sway deflection for the equivalent proportional load case is less than half that of non-proportional type *HV*.

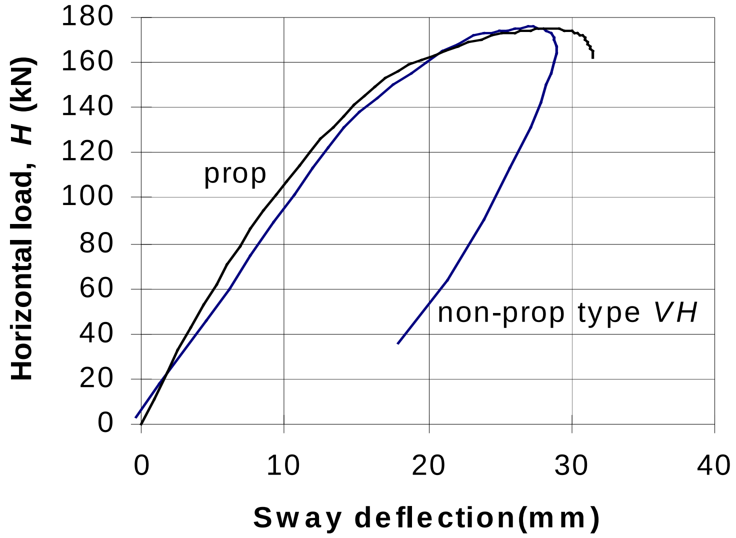

Figure 6: Horizontal load versus sway deflection for case α_{max} = 10

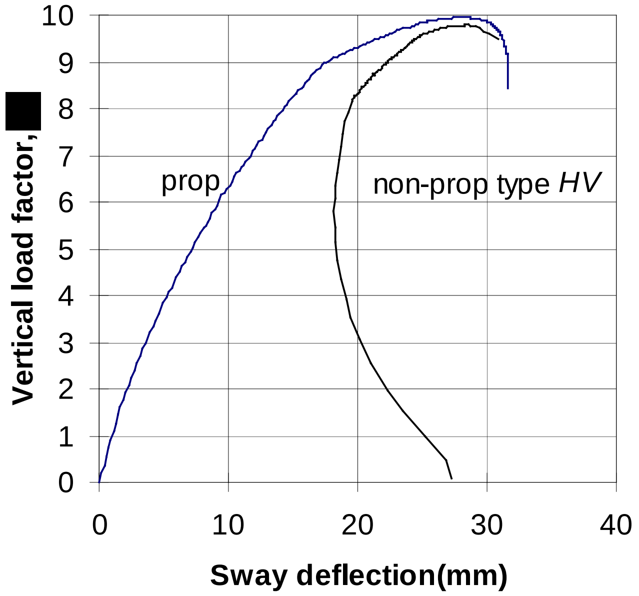

Figure 7: Vertical load factor versus sway deflection for case α_{max} = 10

Another two sets of rectangular portal frames were also analysed by subjecting portal frames to the three different loading types described above. These frames are the same as those described above with only the height been different. The column heights of the second and third set of portal frames are 8m and 12m respectively. Strength interaction diagrams for these two sets of frames are shown in Figures 8 and 9.

The results show that equivalent proportional pattern loads can generally be used to predict the strength of practical concrete portal frames subjected to non-proportional pattern loads.

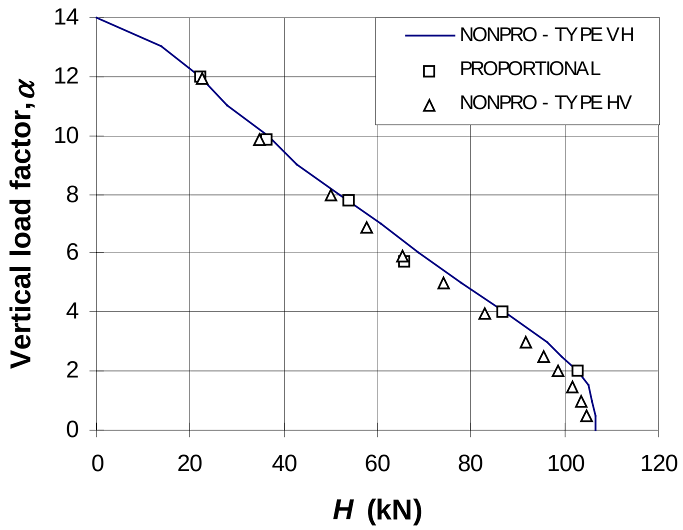

Figure 8: Strength interaction diagram for 8m tall portal frame

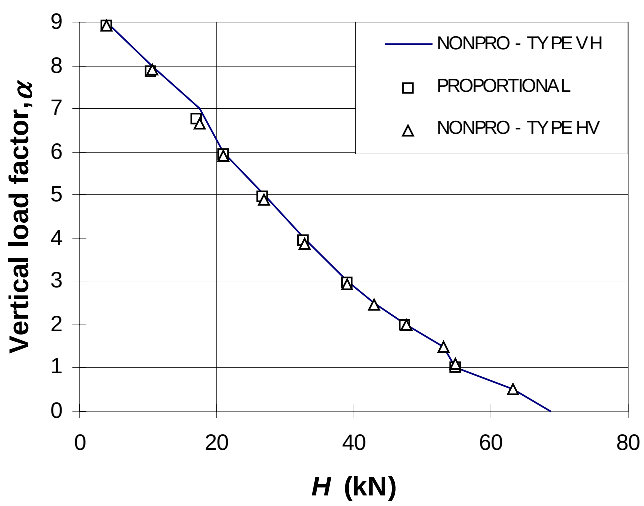

Figure 9: Strength interaction diagram for 12m tall portal frame

5. THREE STOREY FRAME

The behaviour of a three-storey concrete frame under non-proportional loading was investigated. The dimensions of the frame and typical cross sections of its members are shown in Fig. 10. The total reinforcement for each column is 2 percent of its gross cross-sectional area. The total amount of steel in the beams is 2 percent of its own gross cross-sectional area with an equal amount of top and bottom steel. Concrete cover to the centroid of the steel reinforcement is 50mm. The properties of the concrete and reinforcing steel are the same as those used for the column analysed earlier.

The investigation was carried out by first loading each of the storeys using a uniformly distributed load. This was to locate the vertical loads for beam failure at each storey. The peak loads w_{beam} for the first, second and third storey were found to be 45.8 kN/m, 45.8 kN/m and the 43.1 kN/m respectively. Non-proportional pattern load was first applied to the frame. The vertical load pattern w was increased incrementally up to a value of 21.6 kN/m. This corresponds to approximately half the smallest w_{beam} of the storeys. The vertical loads were then maintained at 21.6 kN/m while the horizontal pattern load of H was applied until the latter reached its peak value. This peak value was found to be 34.2 kN.

A proportional loading simulation was subsequently carried out with the ratio of the horizontal load to vertical load maintained as the ratio at the peak load condition for the non-proportional loading case above.

Plots of horizontal load versus sway deflections at the top of the extreme right column is shown in Fig. 11. The plots show that peak load condition of the multi-storey frame under non-proportional loading can be obtained using an equivalent proportional loading. The sway stiffness of the frame was found to be the same at peak load. However, the sway stiffnesses of the frame under non-proportional pattern loads is smaller than that of the same frame under equivalent proportional loads over part of its loading history. This reduction in sway stiffness shows the de-stabilising effect of constant vertical load of $0.5w_{beam}$ for the non-proportional loading case as compared to the gradual increasing vertical load to reach $0.5w_{beam}$ for the proportional loading case.

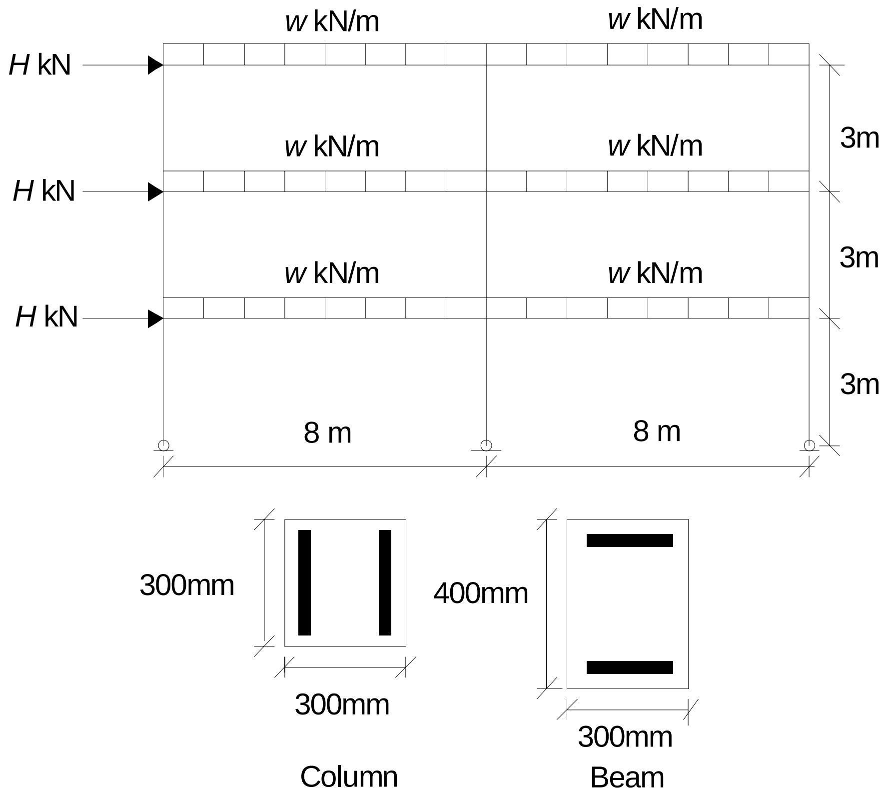

Figure 10: Configuration of 3-storey frame

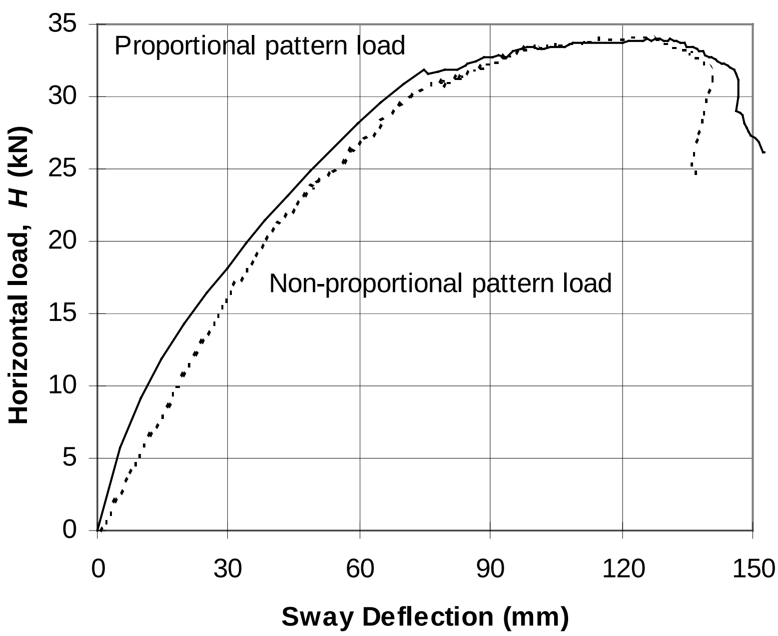

Figure 11: Load versus Sway Deflection Plots for Multi-storey Frames

6. CONCLUDING REMARKS

A curvature-controlled based procedure has been developed which is efficient for tracing the full-range behaviour of non-linear reinforced concrete frames under non-proportional pattern loads. The superiority of the approach is that throughout the entire range, a key-segment curvature-controlled procedure is used. This approach is complementary to an earlier approach developed to study concrete frames under proportional loading. Both are currently being used to study the non-linear behaviour of concrete frames with the ultimate aim of developing a computer-based method for non-linear design of concrete structures.

Load versus deflection relations for columns and frames under non-proportional loading have been obtained. They were compared with those obtained for corresponding frames under equivalent proportional loadings. Results obtained show that an equivalent proportional pattern load may be used to predict the peak loads for a structure subjected to non-proportional load patterns.

Results obtained show that for unbraced practical frames designed to carry predominantly vertical loads, the peak load is virtually independent of the loading paths taken.

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APPENDIX A: CURVATURE CONTROL PROCEDURE FOR CONCRETE FRAMES SUBJECTED TO PROPORTIONAL PATTERN LOAD.

The concrete frame to be analysed is subjected to a unit load pattern. This unit load pattern may be a combination of several vertical and horizontal loads. At any stage during the analysis, the total load is equal to the unit load pattern scaled by a load factor. An example of a frame with a unit load pattern comprising of a vertical udl load of w =1.0 kN/m along the beam and a horizontal load of αwL at beam level, where α = 0.1, is shown in Fig. A1.

The frame is modelled by dividing it into numerous segments and each of these segments is assumed to have uniform flexural stiffness equal to that of the most critically stressed section within it. The section behaviour is modelled by dividing it into numerous concrete and steel layers. The concrete layers are assumed to have a non-linear stress-strain relation, and the steel layers are assumed to have a linearly elastic-plastic stress-strain relation. These stress-strain relations allow for unloading paths due to strain reversal. Structural members such as beams and columns are assumed to have equal size segments. The number of segments chosen to represent each member should give a length to depth ratio of approximately unity.

An incremental-iterative curvature control solution strategy is used. The full range behaviour is obtained by subjecting a nominated key segment to increasing curvatures. This key segment is selected at the start of the analysis. A basis for this selection is that it has monotonically increasing curvatures as the loading process progresses. At the start of a typical curvature-increment step, the targeted curvature for the key segment is calculated by adding a preset curvature increment to the targeted curvature of the preceding step. The load factor consistent with this targeted curvature in the key segment is then determined by carrying out several iterative cycles of calculations until convergence is achieved.

Computational steps required for a typical iterative cycle within an incremental step which has been assigned a targeted curvature K_{step} are :

(1) The frame with the latest full-load stiffnesses EI_i in all the segments is subjected to the unit load pattern. A second order elastic analysis routine is used to determine unit curvatures k_1, k_2,...., k_{key}, k_i, ..., k_{nseg}

in all the segments. A second-order elastic analysis is one which takes into consideration the geometrical nonlinearity effects. If this is the first cycle, then these stiffnesses are set to those calculated at the end of the previous step, else these stiffnesses are set to those calculated at the end of the previous cycle.

(2) The load factor to give the key segment a curvature of K_{step} is then calculated:

$$\lambda = \frac{K_{step}}{\left|k_{key}\right|} \tag{A1}$$

(3) After having calculated λ, it is used to scale the unit curvatures of the segments to give corresponding total curvatures. The term total curvature here means curvature under full loading.

$$K_i = \lambda \times k_i \tag{A2}$$

(4) For each segment *i* with curvatures K_i, a section analysis routine which takes into account the material stress-strain relations is used to calculate the corresponding bending moment, M_i.

The stiffnesses of all the segments are updated:

$$EI_i = \frac{M_i}{K_i} \tag{A3}$$

(5) Convergence of the EI_i values is checked at this stage. If convergence is achieved, a solution has been obtained for the present curvature step. The targeted curvature is then set to that of the next step. Proceed to step (1) to start a new set of iterative cycles. If convergence is not achieved then proceed to step (1) maintaining the present targeted curvature for more iterative cycles.

The solution procedure caters for both geometric and material nonlinearities. The geometrical nonlinearity is taken into consideration when the second order elastic analysis is carried out, and the material non-linearity is taken into consideration when the section analysis is used to update the stiffnesses of the segments.

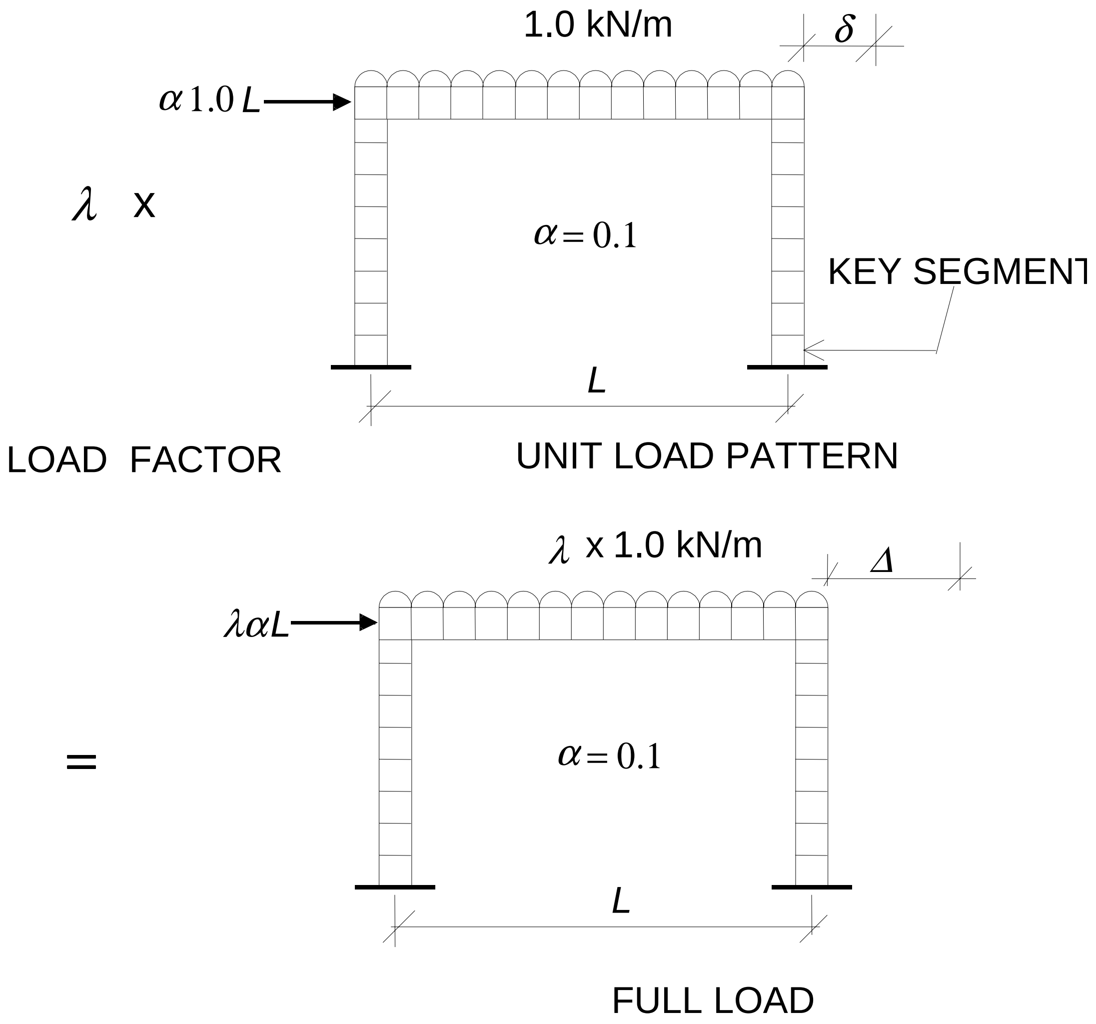

Figure A1: Diagram showing the scaling of the unit load pattern to give full loads.