Non-linear Analysis of Concrete Frames using a Direct Stiffness Line Element Approach

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ABSTRACT: A computationally efficient method of non-linear analysis is described for reinforced concrete frames. The Crisfield arc-length solution procedure is used together with a previously developed line element formulation which takes account of both the material non-linearities and the geometric non-linear effects.

The resulting method has been found to be more reliable and more efficient than the previously-used methods which relied on load-control and displacement-control procedures.

Results obtained from the analysis of several frames are compared with test results reported in the literature.

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1. INTRODUCTION

Concrete structures exhibit distinctly non-linear behaviour, especially at levels of loading close to collapse. Therefore, correct modelling of the material and geometric non-linearities is required if accurate estimates of behaviour at working load and overload are required. The realistic analysis of concrete structures, taking account of non-linear and inelastic behaviour, is tedious and time-consuming. However, over recent years, the increasing availability of high-capacity, low-cost computing facilities has encouraged the development of non-linear analysis procedures (Aas-Jakobson & Grenacher, 1974; Bazant, Pan & Pijaudier-Cabot, 1987; Wong, Yeo and Warner, 1988; Sun, Bradford and Gilbert, 1992,1994; Kawano and Warner, 1995) for use with computers. This and further development of, and improvement to, non-linear analysis procedures in the near future will eventually bring about the introduction and wider use of an accurate non-linear methods of design (Warner, 1993).

Element type, solution procedure and the type of non-linearities which has been considered by various investigators are shown in Table 1 below.

Table 1: Summary of selected procedures for non-linear analysis

Investigators	Year published	Non-linearity	Solution Procedure	Element type
Aas-Jakobson & Grenacher	1974	Geometric & Material	Load control & Displacement control (deflection)	Line element
Crisfield	1983	Geometric & Material	Combined Load & Displacement control (arc-length)	Finite element
Bazant, Pan & Pijaudier-Cabot	1987	Material	Displacement control	Finite element
Wong, Yeo & Warner	1988	Geometric & Material	Displacement control (curvature)	Line element (segmented)
Sun, Bradford & Gilbert	1992, 1994	Geometric & Material	Combined Load & Displacement control (arc-length)	Finite element
Kawano & Warner	1995	Geometric & Material	Load control & Displacement control	Finite element
Wong & Warner (Present method)		Geometric & Material	Combined Load & Displacement control (arc-length)	Line element

Non-linear procedures for the analysis of concrete frames use either a finite element formulation or a direct stiffness line element formulation. This report describes a line element formulation for non-linear analysis of concrete structures.

2. LINE ELEMENT APPROACHES

Direct stiffness line element approaches (Aas-Jakobsen and Grenacher, 1974) discretise structures and their components into numerous axial-flexural beam elements. The property of each element is usually assumed uniform, and dependent on a chosen section along the element which is modelled numerically by dividing the steel and concrete into layers. The axial and flexural stiffnesses are then calculated and these stiffnesses are used to form the element stiffness matrix. In a typical finite-element approach, the structure is also modelled by elements, each with numerous layers. However, the terms of the element stiffness matrix are formed directly from the properties of the layers at a few sections (usually three) along the element (Sun, Bradford and Gilbert, 1992,1994; Kawano and Warner, 1995).

Aas-Jakobsen and Grenacher (1974) described a procedure which uses a geometric non-linear elastic frame analysis as the main routine to analyse concrete frames. Line elements were used to model these structures. Geometric non-linearities within an element were taken into consideration by augmenting each linear-elastic element stiffness k_e with a non-linear geometric element stiffness matrix k_g. Jennings(1968) had earlier pointed out in his paper on modelling of elastic plane frames that using such a formulation to model geometrical non-linearity effects, while including the modification of the element axial stiffness due to axial load, failed to include the modification of axial stiffness due to bowing (ie. lateral deflection). Material non-linearities were included by using a section analysis subroutine which provides flexural and axial stiffnesses of elements based on known thrust and bending moments. Uniform elements with properties equal to those at mid-element were assumed by Aas-Jakobsen et al.

A different line element formulation with segmented elements was previously used by Wong, Yeo and Warner (1988). The properties of the segments making up an element were used to form its element stiffness matrix. This greatly reduces the size of the global stiffness matrix, which resulted in a reduction in both computer memory storage space and program execution time. However, the use of segmented elements, while increasing the computational efficiency, can reduce the accuracy of the modelling of the geometric non-linearities. To enable members with significant bowing, for example compression members, to be modelled accurately, several segmented elements were used to represent each of these members. Geometric non-linearities caused by bowing within the members, with several internal nodes and changing position of loaded nodes, were included by updating the position of nodes during the analysis. Thus, members which were least affected by geometric non-linearities, e.g. beams, were modelled using a single segmented element.

Both the line element procedures of Aas-Jakobsen et al. and Wong et al. described above fail to model the geometric non-linearities accurately unless a large number of elements are used per member. This report describes a line element procedure which can model accurately the geometric non-linearities present in a frame.

3. PROPOSED METHOD OF ANALYSIS

The development of the computer program has been undertaken in two distinct stages. The first stage involved the development of a program capable of giving accurate solutions to geometrically non-linear plane frames. While binary-coded programs capable of analysing geometrically non-linear frames are available commercially, the independent development of such a program enabled computer source codes to be created for subsequent modification to include the material nonlinearity effects. After having ascertained that the developed geometric non-linear analysis program was accurate, by comparing results with those obtained from published analytic solutions (Frish-Fay, 1962; Lee et al., 1968), the second stage of adding the appropriate subroutines for material non-linearities was carried out. Sections 3.1 and 3.2 below describe the program in accordance to these two stages of development.

3.1 Program for Analysing Second-order Elastic Frames

In developing a procedure for the analysis of geometrically non-linear frames, the line element formulation of Jennings (1968) was used, together with the arc-length solution method of Crisfield (1981,1983).

Jennings' line element formulation allows for significant change in geometry under loading. The same formulation was also been used by others (Meek and Tan, 1983; Meek, 1991). With this formulation, it is possible to determine the behaviour of a plane frame with linear-elastic material properties until its deformed shape bears little resemblance to its original configuration (Jennings, 1968; Meek and Tan, 1983).

This formulation includes two parts which make it well-suited for use with a predictor-corrector solution procedure such as that of Crisfield. These two parts are (1) a tangent structural stiffness matrix which allows the prediction of an incremental load scaling factor and the corresponding displacements of the frame, and (2) a secant formulation which allows element forces to be determined based on the total displacements in the structure predicted using the tangent formulation of (1). Part (1) forms the predictor and part (2) forms the corrector for use after each prediction. A predictor-corrector solution procedure has also been adopted by Kawano and Warner (1995).

In other words, for each iterative cycle the structural tangent stiffness is used to predict the incremental displacements to obtain the latest position of the structure at the end of the iterative cycle, and the structural secant formulation is used to determine the forces required at the nodes to maintain the frame at this position. After having determined these forces, the out-of-balanced forces are determined. These out-of-balance forces are used for the next iterative cycle to improve the accuracy of the solution until the solution converges to within an acceptable tolerance.

Jennings' formulation takes into consideration all geometrical nonlinearities present in a structure and solutions obtained have been found to predict the behaviour of elastic structures accurately (Jennings, 1968). The geometrically non-linearity effects considered in this formulation are:

- the change of element lateral stiffness,
- the finite deflection of joints, and
- the change of element length due to bowing.

Jennings' matrix formulation will not be presented here. It is described elsewhere (Jennings, 1968; Meek, 1991).

3.2 Modification of program to include modelling for material non-linearities

Jennings' line element formulation accurately predicts the behaviour of geometrically non-linear frames and is most suitable for use as the foundation for the non-linear analysis of concrete structures because accurate modelling of the geometrical non-linearities is assured.

Solution procedures for non-linear structural analysis problems can be based on load control, displacement control or combined load-displacement control. Load control procedures, while useful for structures subjected to working loads, are not suitable for tracing behaviour at collapse. Using a displacement control procedure can be inefficient as the control displacement parameter needs to be decided before the commencement of the analysis. The control displacement parameter must be able to act as a monitor for the stiffness degradation of the chosen structure at all stages of loading, irrespective of where in the structure this degradation occurs. For example, for a free-standing cantilever column, the degradation of stiffness, indicated by the horizontal deflection at the top of the column, is mainly affected by the deformation of the segment at the base. Therefore, the curvature of this segment is suitable for use as the control displacement parameter. However, it is difficult to use a purely displacement control procedure at one location, e.g. a curvature or a deflection, for complicated frames where degradations occurring almost simultaneously in several regions.

For example, the yielding in a beam forming part of a multi-storey building does not affect the sway deflection as much as the yielding in a column. Choosing an unsuitable control displacement results in non-convergence of the solution procedure. For such cases, an arc-length control procedure can be used.

The arc-length control procedure, which is based on a control parameter in the form of a constant 'length' in a multi-dimensional load-displacement space. For a structure with many degrees of freedom of movement, load-displacement control can be simultaneously carried out at all the degrees-of-freedom at each node. It is, therefore, more efficient for sensing the occurrence of multiple non-linear events, such as simultaneous yielding in different parts of a structure. Hence the arc-length solution procedure was selected for use in the present program, to trace non-linear behaviour, up to, and beyond, the point of collapse. Details of the arc-length procedure are given in Appendix A. In the present method, the tangent stiffness is updated at the start of every iterative cycle rather than at the start of the incremental step shown in Figure A1.

Jennings (1968) mentioned in his paper that a program had been developed to include the effect of material non-linearities using his formulation. However, the details of this program were not given. It is not known whether this program was developed to analyse concrete structures.

Sun et al. (1992, 1994) describe the development of a program which also uses the arc-length solution procedure of Crisfield. This program uses a finite element formulation instead of the direct stiffness line element formulation used in the present approach. A program developed by Kawano and Warner (1995) uses a finite element method similar to that by Sun et al. but it includes time-dependent effects and utilises a displacement control solution procedure.

The procedure to analyse non-linear concrete frames is illustrated using the flow-diagram shown in Figure 1. Terms shown in this figure are the same as those given in Appendix A. This figure shows diagrammatically the inclusion of the section analysis subroutine. This section analysis routine models sections by using steel and concrete layers. The stress-strain relation used is that first described by Warner (1969), with the concrete in tension having a relation described by Kenyon and Warner (1993). Unloading paths are included for concrete in both compression and tension. Concrete in tension is assumed to unload towards the origin, and concrete in compression is assumed to unload with initial slope. This is shown in Figure 2. The stress-strain relation for steel reinforcement is assumed to be elastic-plastic; steel reinforcements in both tension and compression are assumed to unload with initial slope.

3.3 Advantages of the Present Approach

The present approach ensures that the geometrical non-linearities are taken into consideration accurately in a non-linear frame. Premature buckling for a frame with compression members can thus be predicted accurately.

The use of uniform property elements enables finite length hinges to develop in the structure during the analysis. The present approach assumes that the property of an element is uniform, equal to that of the most critically stressed end-section. This assumption is conservative for most elements. Previous researchers such as Bazant (1976) and Bazant, Pan and Pijaudier-Cabot (1987) showed that hinges in concrete structures are of finite length. Bazant et al. (1987) suggested that the length of hinges should be approximately equal to its depth.

4. COMPARISONS WITH TEST RESULTS

In order to check the accuracy of the present approach, theoretical results have been obtained for some columns and frames used in previous experimental studies. The following material properties are assumed in the analysis. Modulus of elasticity for steel E_s is taken as 2.0E5 MPa and that of concrete E_c as $5050\sqrt{f_{cm}}$ MPa, where f_{cm} is the mean concrete strength. The mean in-situ concrete strength is assumed to be equal to the mean cylinder strength. Value of the strain at maximum stress ε_{cmax} is assumed to be 0.002. The parameter γ_2 used to define the shape of the concrete stress-strain curve (Warner,1969) is assumed to be 3.0. Where strength of concrete was determined using cubes, the conversion of f_{cm} = 0.8 f_{cube} was used.

4.1 Short Columns

Short columns subjected to concentric and eccentric loadings were tested by Hognestad (1951) and Hudson (1965). Results from these tests, and predictions from the present analysis are summarised in Table 2 and Table 3, respectively. The ratios of the test to calculated ultimate loads for the columns tested by Hognestad and Hudson are listed in the last columns of these tables. Members with concentric load were analysed by assuming a very small load eccentricity of 1.0 mm.

The average and the standard deviation for P_{test}/P_{calc} obtained for Hognestad's test columns are 0.94 and 0.07, and those for Hudson's test columns are 1.04 and 0.07.

Table 2: Hognestad's tests of short columns

Tested by	Slender-ness l/d	Speci-men	Concrete strength, f_{cm} (MPa)	Cross-Section Width (mm)	Cross-Section Depth,d (mm)	Ecc. ratio e/d	P_{test} (kN)	P_{calc} (kN)	P_{test} / P_{calc}
Hognestad (1956) group II	7.5	B-6a	28.1	254	254	0.00	2028	1967	1.03
	7.5	B-6b	27.9	254	254	0.00	1868	1989	0.94
	7.5	C-6a	13.9	254	254	0.00	1001	1218	0.82
	7.5	C-6b	10.5	254	254	0.00	898	1052	0.85
	7.5	A-7a	36.1	254	254	0.33	1219	1168	1.04
	7.5	A-7b	40.1	254	254	0.25	1263	1491	0.85
	7.5	B-7a	28.1	254	254	0.25	1139	1157	0.98
	7.5	B-7b	27.9	254	254	0.25	1103	1153	0.96
	7.5	C-7a	13.6	254	254	0.25	627	731	0.86
	7.5	C-7b	10.5	254	254	0.25	564	638	0.88
	7.5	A-8a	38.1	254	254	0.50	721	823	0.88
	7.5	A-8b	40.1	254	254	0.50	676	842	0.80
	7.5	B-8a	32.4	254	254	0.50	694	764	0.91
	7.5	B-8b	29.4	254	254	0.50	649	730	0.89
	7.5	C-8a	12.5	254	254	0.50	440	463	0.95
	7.5	C-8b	12.5	254	254	0.50	440	463	0.95
	7.5	A-9a	35.2	254	254	0.75	396	439	0.90
	7.5	A-9b	35.6	254	254	0.75	406	441	0.92
	7.5	B-9a	32.4	254	254	0.75	418	433	0.97
	7.5	B-9b	30.1	254	254	0.75	398	427	0.93
	7.5	C-9a	13.0	254	254	0.75	325	349	0.93
	7.5	C-9b	11.9	254	254	0.75	291	338	0.86
	7.5	A-10a	35.2	254	254	1.25	205	209	0.98
	7.5	A-10b	35.6	254	254	1.25	196	209	0.94
	7.5	B-10a	29.4	254	254	1.25	193	205	0.94
	7.5	B-10b	30.1	254	254	1.25	196	206	0.95
	7.5	C-10a	15.9	254	254	1.25	198	194	1.02
	7.5	C-10b	12.2	254	254	1.25	200	191	1.05

Table 2 -contd : Hognestad's tests of short columns

Tested by	Slenderness l/d	Specimen	Concrete strength, f_{cm} (MPa)	Cross-Section Width (mm)	Cross-Section Depth,d (mm)	Ecc. ratio e/d	P_{test} (kN)	P_{calc} (kN)	P_{test} / P_{calc}
Hognestad (1956) group III	7.5	B-11a	26.7	254	254	0.00	2224	2436	0.91
	7.5	B-11b	27.7	254	254	0.00	2157	2485	0.87
	7.5	C-11b	14.3	254	254	0.00	1570	1758	0.89
	7.5	A-12a	28.6	254	254	0.25	1401	1419	0.99
	7.5	A-12b	34.8	254	254	0.25	1446	1596	0.91
	7.5	B-12a	29.7	254	254	0.25	1348	1447	0.93
	7.5	B-12b	27.7	254	254	0.25	1263	1388	0.91
	7.5	C-12a	15.9	254	254	0.25	1121	1048	1.07
	7.5	C-12b	15.2	254	254	0.25	1023	1026	1.00
	7.5	A-13a	36.9	254	254	0.50	979	1070	0.91
	7.5	A-13b	33.4	254	254	0.50	934	1012	0.92
	7.5	B-13a	24.7	254	254	0.50	801	861	0.93
	7.5	B-13b	29.6	254	254	0.50	916	946	0.97
	7.5	C-13a	15.9	254	254	0.50	672	699	0.96
	7.5	C-13b	14.3	254	254	0.50	609	671	0.91
	7.5	A-14a	36.9	254	254	0.75	632	720	0.88
	7.5	A-14b	35.2	254	254	0.75	681	712	0.96
	7.5	B-14a	24.7	254	254	0.75	617	640	0.96
	7.5	B-14b	31.7	254	254	0.75	489	691	0.71
	7.5	C-14a	13.5	254	254	0.75	514	498	1.03
	7.5	C-14b	14.3	254	254	0.75	463	511	0.91
	7.5	A-15a	35.2	254	254	1.25	391	365	1.07
	7.5	A-15b	33.4	254	254	1.25	351	364	0.97
	7.5	B-15a	26.2	254	254	1.25	329	357	0.92
	7.5	B-15b	31.9	254	254	1.25	376	362	1.04
	7.5	C-15a	13.5	254	254	1.25	322	330	0.98
	7.5	C-15b	14.3	254	254	1.25	331	334	0.99
							Average		0.94
							Std Deviation		0.07

Table 3: Hudson's tests of short columns

Tested by	Slender-ness l/d	Speci-men	Concrete strength, f_{cm} (MPa)	Cross-Section Width (mm)	Cross-Section Depth,d (mm)	Ecc. ratio e/d	P_{test} (kN)	P_{calc} (kN)	P_{test} / P_{calc}
Hudson (1956) Series I	8	11	24.8	102	102	0.00	267	272	0.98
	8	12	24.8	102	102	0.00	267	272	0.98
	8	13	24.8	102	102	0.00	299	272	1.10
	8	14	24.8	102	102	0.00	264	272	0.97
	8	21	26.9	102	102	0.00	311	300	1.04
	8	22	26.9	102	102	0.00	289	300	0.96
	8	23	26.9	102	102	0.00	309	300	1.03
	8	24	26.9	102	102	0.00	311	300	1.04
	8	31	28.3	102	102	0.00	307	306	1.00
	8	32	28.3	102	102	0.00	311	306	1.02
	8	33	28.3	102	102	0.00	289	306	0.94
	8	34	28.3	102	102	0.00	288	306	0.94
	8	41	25.5	102	102	0.00	289	279	1.04
	8	42	25.5	102	102	0.00	306	279	1.10
	8	43	25.5	102	102	0.00	306	279	1.10
	8	44	25.5	102	102	0.00	307	279	1.10
Series II	8	11	24.8	102	102	0.30	156	157	0.99
	8	12	24.8	102	102	0.30	196	157	1.25
	8	13	24.8	102	102	0.30	165	157	1.05
	8	14	24.8	102	102	0.30	178	157	1.13
	8	21	26.9	102	102	0.30	165	169	0.97
	8	22	26.9	102	102	0.30	196	169	1.16
	8	23	26.9	102	102	0.30	173	169	1.03
	8	24	26.9	102	102	0.30	173	169	1.03
	8	31	28.3	102	102	0.30	200	179	1.12
	8	32	28.3	102	102	0.30	200	179	1.12
	8	33	28.3	102	102	0.30	178	179	0.99
	8	34	28.3	102	102	0.30	169	179	0.94
	8	41	25.5	102	102	0.30	200	196	1.02
	8	42	25.5	102	102	0.30	200	196	1.02
	8	43	25.5	102	102	0.30	200	196	1.02
	8	44	25.5	102	102	0.30	187	196	0.95
							Average		1.04
							Std Deviation		0.07

4.2 Long Columns

The results from the analysis of long columns are compared with test data in the last column of Tables 4 through 11. The average and standard deviation of the ratios of P_{test}/P_{calc} are also given in these tables. The concrete mean strength was assumed equal to the average cylinder strength. The reasonably good agreement for the individual investigation and the absence of any definite trends with major variables such as slenderness, load eccentricity, material properties suggests that the ultimate load of hinged columns can be predicted with good accuracy using the present analysis. Members with concentric load were analysed by assuming a small load eccentricity.

Table 4 : Columns tested by Breen and Ferguson (1969)

Speci-men	Slender-ness L/h	Concrete strength, f_{cm} (MPa)	Cross-Section Width (mm)	Cross-Section Depth,h (mm)	Ecc. ratio e_2 /h	P_{test} (kN)	P_{calc} (kN)	P_{test} / P_{calc}
G1	20	25.6	152	102	0.30	151	169	0.89
G2	40	25.2	152	102	0.60	48	47	1.02
G3	50	25.5	152	102	0.75	30	31	0.97
G4	50	25.5	152	102	0.30	53	52	1.03
G5	60	28.7	152	102	0.90	29	23	1.28
G6	50	30.2	152	102	0.30	49	48	1.02
G7	40	33.4	152	102	0.20	67	76	0.88
G8	60	28.0	152	102	0.27	48	50	0.96
G9	20	27.4	152	102	0.30	147	164	0.90
G10	10	27.7	152	102	0.30	209	230	0.91
						Average		0.98
						Std Deviation		0.12

Table 5 : Columns tested by Chang and Ferguson (1963)

Speci-men	Slender-ness L/h	Concrete strength, fcm (MPa)	Cross-Section Width (mm)	Cross-Section Depth,h (mm)	Ecc. ratio e2 /h	P_{test} (kN)	P_{calc} (kN)	P_{test} / P_{calc}
1	31	23.3	156	103	0.07	168	187	0.90
2	31	35.0	156	103	0.39	69	84	0.82
3	31	28.9	156	103	0.06	189	229	0.83
4	31	30.1	156	103	0.38	73	80	0.91
5	31	32.8	156	103	0.21	123	132	0.93
6	31	33.6	156	103	0.06	197	250	0.79
						Average		0.86
						Std Deviation		0.06

Table 6: Columns tested by Martin and Olivieri (1965)

Speci-men	Slender-ness L/h	Concrete strength, f_{cm} (MPa)	Cross-Section Width (mm)	Depth,h (mm)	Ecc. ratio e_2/h	P_{test} (kN)	P_{calc} (kN)	P_{test} / P_{calc}
402-1	40.0	30.0	127	90	0.00	147	157	0.93
402-2	40.0	24.3	127	90	0.00	125	137	0.91
412-1	40.0	33.6	127	90	0.21	118	123	0.96
412-2	40.0	25.0	127	90	0.21	89	102	0.87
422-1	40.0	34.9	127	90	0.39	93	89	1.05
422-2	40.0	25.7	127	90	0.39	76	76	0.99
432-1	40.0	37.3	127	90	0.28	96	113	0.85
432-2	40.0	26.4	127	90	0.28	93	93	1.00
						Average		0.95
						Std Deviation		0.07

Note: Columns with double curvature e_1/e_2 = - 0.5

Table 7: Columns tested by Thomas (1939)

Speci-men	Slender-ness L/h	Concrete strength, f_{cm} (MPa)	Cross-Section Width (mm)	Depth,h (mm)	Ecc. ratio e_2/h	P_{test} (kN)	P_{calc} (kN)	P_{test} / P_{calc}
LC1	14.75	24.27	152.4	152.4	0.00	588	554	1.06
LC2	20.75	24.27	152.4	152.4	0.00	545	606	0.90
LC3	23.75	24.27	152.4	152.4	0.01	478	498	0.96
LC4	26.75	24.27	152.4	152.4	0.01	465	439	1.06
LC5	26.75	24.27	152.4	152.4	0.05	456	363	1.26
LC6	23.75	24.27	152.4	152.4	0.04	448	451	0.99
LC7	20.75	24.27	152.4	152.4	0.04	463	493	0.94
LC8	14.75	24.27	152.4	152.4	0.03	474	531	0.89
LC9R	26.75	24.27	152.4	152.4	0.02	360	360	1.00
LC10	23.75	24.27	152.4	152.4	0.04	374	374	1.00
LC11	20.75	24.27	152.4	152.4	0.04	418	379	1.10
LC12	14.75	24.27	152.4	152.4	0.03	438	492	0.89
PLC1	33.1667	24.27	76.2	76.2	0.06	82	62	1.32
PLC2	33.1667	24.27	76.2	76.2	0.06	81	63	1.28
						Average		1.05
						Std Deviation		0.14

Table 8: Columns tested by Green and Hellesland (1975)

Speci-men	Slender-ness L/h	Concrete strength, f_{cm} (MPa)	Cross-Section Width (mm)	Depth,h (mm)	Ecc. ratio e_2/h	P_{test} (kN)	P_{calc} (kN)	P_{test}/P_{calc}
S1	15	34.4	178	127	0.10	502	556	0.90
S5	15	33.6	178	127	0.09	621	707	0.88
						Average		0.89
						Std Deviation		0.02

Table 9: Columns tested by MacGregor and Barter (1965)

Speci-men	Slender-ness	Concrete strength, f_{cm} (MPa)	Cross-Section Width (mm)	Depth,h (mm)	Ecc. ratio e_2/h	P_{test} (kN)	P_{calc} (kN)	P_{test}/P_{calc}
A1	27	33.6	112	64	0.20	169	185	0.91
A2	27	32.7	112	64	0.20	169	186	0.91
B1	27	29.0	112	64	1.50	33	30	1.10
B2	27	32.6	112	64	1.50	31	30	1.05
						Average		0.99
						Std Deviation		0.10

note: Columns bent in double curvature, $e_1/e_2 = -1$

Table 10: Columns tested by Drysdale and Huggins(1971)

Speci-men	Slender-ness	Concrete strength, f_{cm} (MPa)	Cross-Section Width (mm)	Depth,h (mm)	Ecc. ratio e_2/h	P_{test} (kN)	P_{calc} (kN)	P_{test}/P_{calc}
D-1-A	31	30.3	127	127	0.20	173	165	1.05
D-1-B	31	30.3	127	127	0.20	172	165	1.04
D-2-C	31	29.2	127	127	0.20	177	163	1.08
D-2-D	31	29.2	127	127	0.20	180	163	1.11
						Average		1.07
						Std Deviation		0.03

Table 11: Columns tested by Absel-Sayed and Gardner (1975)

Speci-men	Slender-ness	Concrete strength, f_{cm} (MPa)	Cross-Section Width (mm)	Depth,h (mm)	Ecc. ratio e_2/h	P_{test} (kN)	P_{calc} (kN)	P_{test}/P_{calc}
D1	23	30.8	150	150	0.423	127	144	0.88
D4	23	31.2	150	150	0.847	62	71	0.89
D8	23	31.0	150	150	1.267	44	46	0.95
						Average		0.91
						Std Deviation		0.04

4.3 Frames

The results from the analysis of simple rectangular frames by the present method and the analysis carried out by Chang(1967) are compared with test data in Table 12 for the test frames shown in Figure 3. Chang used an inelastic analysis which was applicable to simple box frames. The analysis took into consideration both material and geometric non-linearities. For the present approach, the ratio P_{test}/P_{calc} ranges from 0.82 to 1.18 with an average and a standard deviation of 0.96 and 0.14 respectively. From the approach by Chang(1967) for simple frames, the ratio of P_{test}/P_{calc} obtained by him ranges from 0.73 to 1.19 with an average and a standard deviation of 0.98 and 0.15 respectively.

The differences between the values from tests and those obtained from the present analysis may be due to any of the following:

- Variations between the mean test strengths and the in-situ material strengths. In the present analysis, they were assumed to be the same.
- Joint effects on the test loads.
- Un-intentional restraining effect from the applied loads.

Therefore, comparisons between results from the present analysis with those obtained by the analysis by Chang(1967) were carried out. The average and standard deviation for the ratio of the ultimate load calculated by Chang to the ultimate load calculated by the present method are 0.99 and 0.09. The standard deviation is smaller than the value of 0.14 obtained earlier for P_{test}/P_{calc} for the present method. While the analysis by Chang gave results comparable to those obtained using the present analysis, the present analysis has the advantage of not been restrictive in its application.

5. CONCLUDING REMARKS

An efficient approach for the non-linear analysis of reinforced concrete frames using line elements has been developed. This approach was developed by including a routine for analysing material non-linearity into a geometric non-linear solution procedure which uses an accurate, existing line element formulation presently used for the analysis of geometrically non-linear frames.

Table 12: Test results versus calculated values for rectangular frames

Frame	width/ height	P_{test} (kN)	Present Method		Results by Chang (1967)		P_{calc}(Chang) / P_{calc} (Present Method)
			P_{calc} (kN)	P_{test} / P_{calc}	P_{calc}(Chang) (kN)	P_{test} / P_{calc}	
Frames tested by Furlong and Ferguson (1965)							
F-1	21	267	296	0.90	294	0.91	0.99
F-2R	21	274	333	0.82	298	0.92	0.89
F-3R	21	177	172	1.03	154	1.15	0.90
F-4	21	234	224	1.04	211	1.11	0.94
F-5	16	247	276	0.89	241	1.02	0.87
F-6	16	200	171	1.17	184	1.09	1.08
Frames tested by Breen and Ferguson (1964)							
F-1	30	262	222	1.18	227	1.15	1.02
F-2	30	262	287	0.91	274	0.96	0.95
F-3	15	271	250	1.08	227	1.19	0.91
F-4	15	371	361	1.03	316	1.17	0.88
Frames tested by tested by Ferguson and Breen (1965)							
L-1	20	167	225	0.74	217	0.77	0.96
L-2	20	111	121	0.92	139	0.80	1.15
L-3	20	138	143	0.97	145	0.95	1.01
L-5	10	189	170	1.11	191	0.99	1.12
L-6	10	245	287	0.85	299	0.82	1.04
L-7	10	178	237	0.75	245	0.73	1.03
Average				0.96		0.98	0.99
Std dev.				0.14		0.15	0.09

The resulting approach allows the modelling of frames which include the formation of finite length hinges in region of strength degradation.

Comparison of the collapse loads obtained from the present analysis with tests reported in the literature shows that the present analysis gives good estimates of collapse loads.

The approach is a suitable tool for use with non-linear design method. It is at present being used to determine the safety requirements associated with using such an analytical approach for non-linear design. A preliminary report of this work has been published (Wong and Warner, 1997).

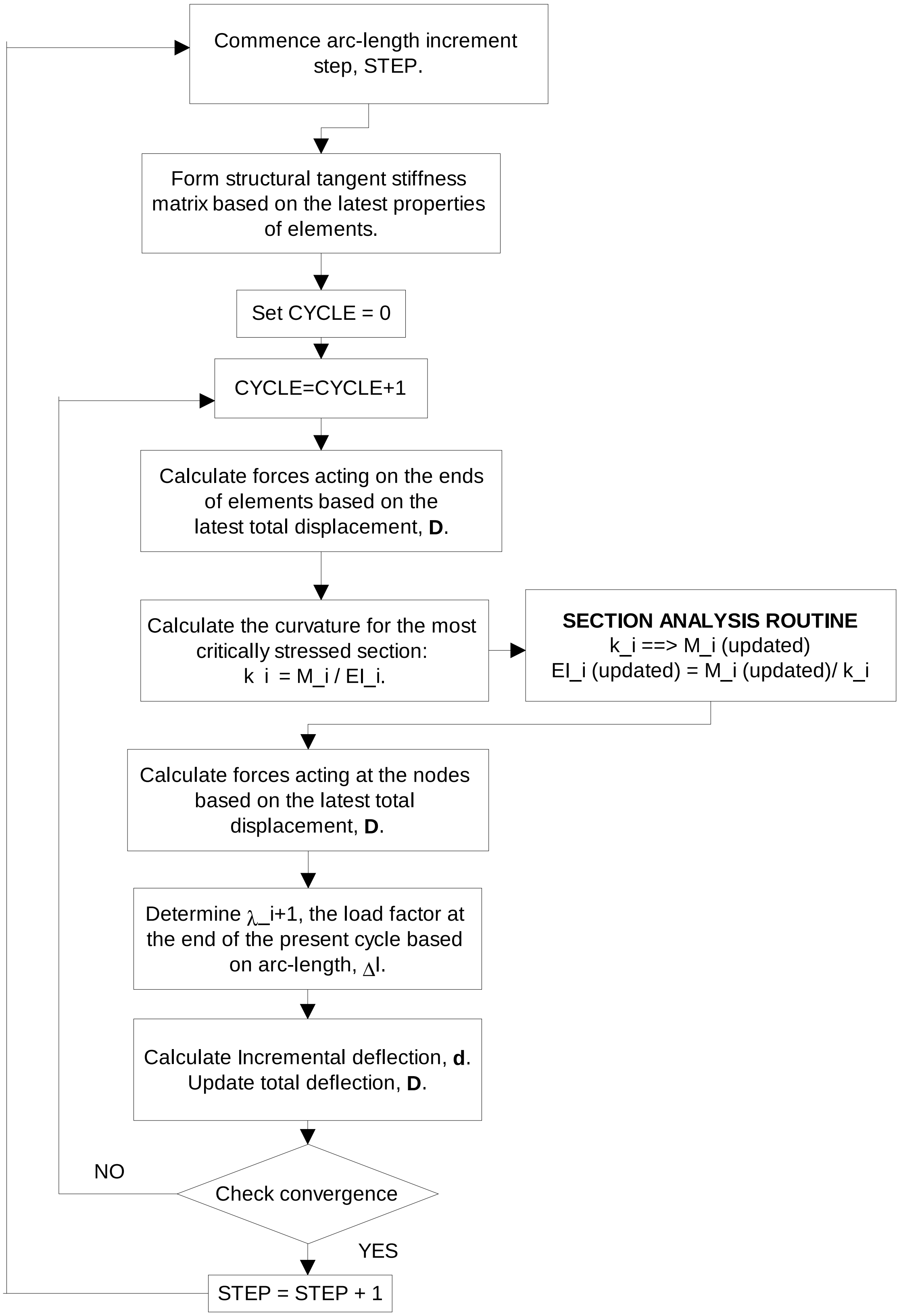

Fig 1: Calculation cycle for a typical step

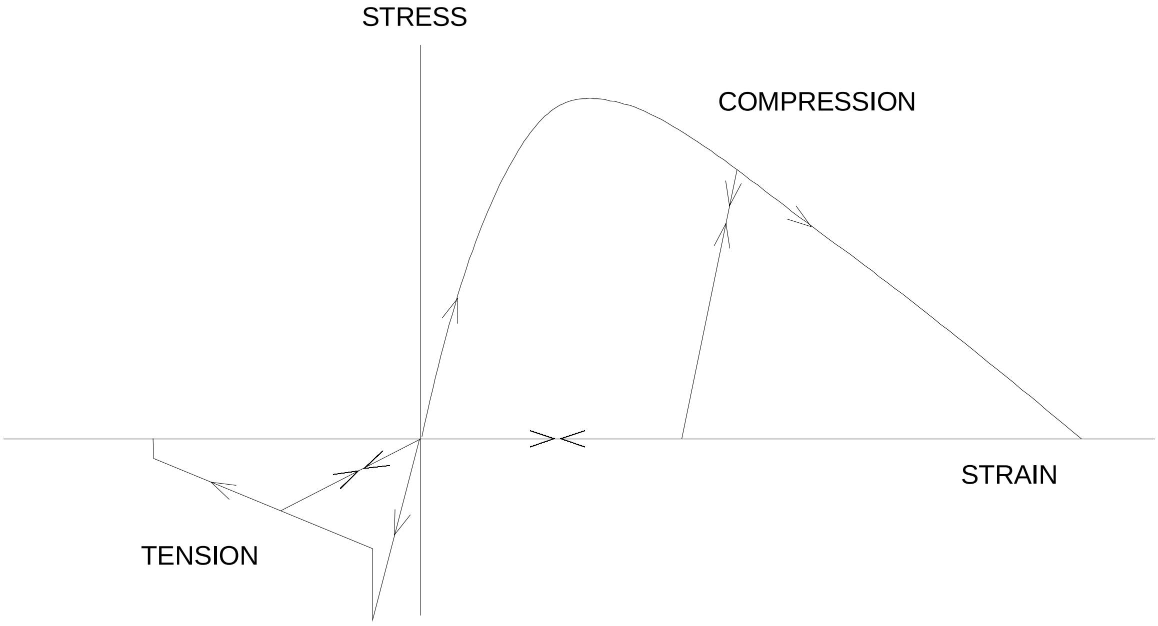

Fig 2: Diagram showing unloading paths for concrete

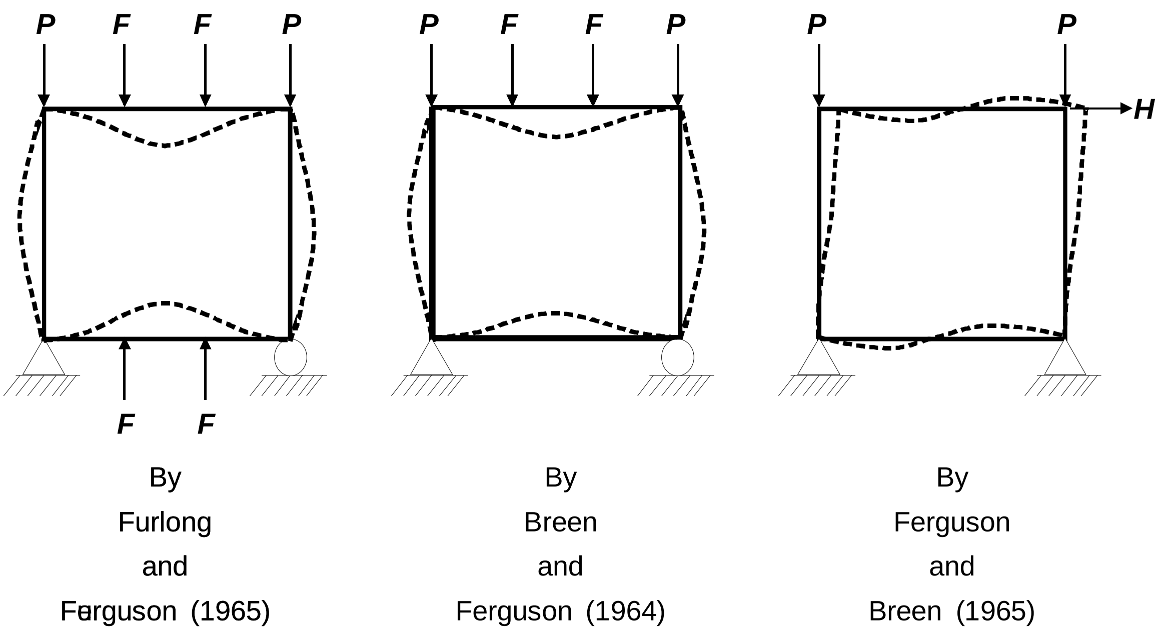

Fig 3: Rectangular test frames (after Chang, 1967)

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APPENDIX A: CRISFIELD'S ARC-LENGTH SOLUTION PROCEDURE

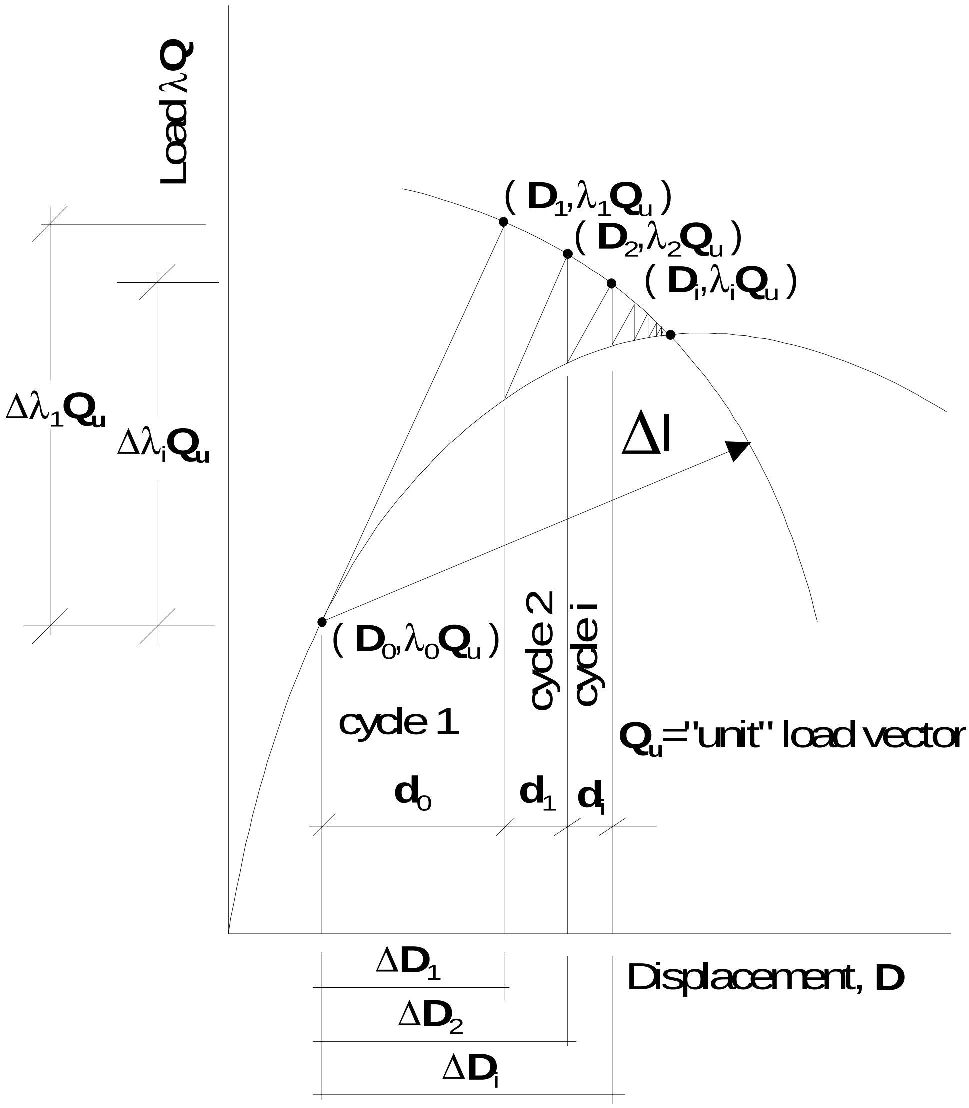

Fig. A1: The arc-length solution procedure (Crisfield, 1983)

This appendix describes Crisfield's arc-length solution procedure (Crisfield, 1983).

Figure A1 shows that for a typical iterative cycle *i*, the incremental displacement $\mathbf{d}_i$ is calculated by:

$$\begin{aligned}\mathbf{d}_i &= \mathbf{K}_t^{-1}\left[f(\mathbf{D}_i) - \lambda_{i+1}\mathbf{Q}_u\right] \\ &= -\mathbf{K}_t^{-1} f(\mathbf{D}_i) + \lambda_{i+1}\mathbf{K}_t^{-1}\mathbf{Q}_u \\ &= \mathbf{d}_{Ni} + \lambda_{i+1}\mathbf{d}_T\end{aligned}$$

where $\mathbf{d}_{Ni} = -\mathbf{K}_t^{-1} f(\mathbf{D}_i)$ and $f(\mathbf{D}_i)$ is the nodal forces based on the total displacements at the end of iterative cycle *i*.

At the beginning of the step, $\mathbf{d}_T = -\mathbf{K}_t^{-1}\mathbf{Q}_u$ and $\mathbf{d}_{Ni}$ are known. To obtain $\mathbf{d}_i$ for each cycle, λ_{i+1} needs to be determined.

The arc-length constraint equation to determine λ_{i+1} is

$$a_1\lambda_{i+1}^2 + a_2\lambda_{i+1} + a_3 = 0$$

where

$$a_1 = \mathbf{d}_T^T\mathbf{d}_T$$

$$a_2 = 2\mathrm{d}_1 + 2\mathrm{d}_2$$

$$a_3 = \mathrm{d}_3 + 2\mathrm{d}_4 + \left[\Delta\mathbf{D}_i^T\Delta\mathbf{D}_i - \Delta l^2\right]$$

$\mathrm{d}_1 = \mathbf{d}_T^T\Delta\mathbf{D}_i$, $\mathrm{d}_2 = \mathbf{d}_T^T\mathbf{d}_{Ni}$, $\mathrm{d}_3 = \mathbf{d}_{Ni}^T\mathbf{d}_{Ni}$ and $\mathrm{d}_4 = \mathbf{d}_{Ni}^T\Delta\mathbf{D}_i$.

Solving the constraint equation above gives two values of λ_{i+1}; and substituting these values into the equation below gives two corresponding values of $\cos\theta$, say *c1* and *c2*.

$$\cos\theta = 1 + \frac{1}{\Delta l^2}\left(d_4 + \lambda_{i+1}d_1\right)$$

If one of *c1* and *c2* is positive, choose λ_{i+1} corresponding to the one which is positive. However, if both are positive, choose λ_{i+1} closest to $\lambda_{i+1,lin}$ given below:

$$\lambda_{i+1,lin} = -\frac{a_3}{a_2}$$

The arc-length Δl is based on an initial guess of $\Delta\lambda$,and is obtained from the expression:

$$\Delta\lambda = a\frac{\Delta l}{\sqrt{\mathbf{d}_T^T\mathbf{d}_T}}$$

where a is the sign of r and $r = \mathbf{d}_T^T\mathbf{K}_t\mathbf{d}_T$.