

**Non-linear Design of Continuous Reinforced Concrete
Flexural Members**

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K W Wong

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ABSTRACT: The methods of strength design specified in most current design standards for concrete structures concentrate on the strength of individual cross sections and use linear methods of analysis to evaluate stress resultants.

Nevertheless, in recognition of the highly non-linear mode of behaviour at full working load and at collapse, provision is made in many design standards for the alternative of direct collapse load design based on non-linear analysis. Before non-linear design methods can be implemented, reliable safety coefficients need to be evaluated.

In this report a back-calibration method is suggested for evaluating a system safety coefficient which can be used in the direct non-linear design of concrete structures. This coefficient takes into account of the ϕ factor in ultimate strength design, plus the use of mean material properties, and the use of non-linear analysis in lieu of linear elastic analysis. Values of the system safety coefficient are derived for indeterminate concrete beams. The system safety coefficient has been found to be quite insensitive to both the support condition and the strength grade of concrete.

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1. NON-LINEAR ANALYSIS AND DESIGN

The methods of strength design specified in most current design standards concentrate on the strength of individual cross sections and use linear methods of analysis to evaluate stress resultants.

Nevertheless, in recognition of the highly non-linear mode of behaviour at full working load and at collapse, provision is made in many design standards for the alternative of direct collapse load design based on non-linear analysis. Before non-linear design methods can be implemented, reliable safety coefficients need to be evaluated.

In this report a suitable back-calibration method is suggested for evaluating a system safety coefficient which can be used in the direct non-linear design of concrete structures. This coefficient takes into account of the ϕ factor in ultimate strength design, plus the use of mean material properties, and the use of non-linear analysis in lieu of linear elastic analysis. Values of the system safety coefficient are derived for continuous concrete beams.

2. EVALUATION OF SYSTEM SAFETY COEFFICIENT BY BACK-CALIBRATION

For a continuous beam designed in accordance with AS 3600 (Standard Australia, 1994) with a chosen moment redistribution factor β , simple equilibrium relations exist between the design ultimate moments in the negative and positive bending moment regions and the design ultimate load. Taking, for example, the simple case of a beam with fully fixed ends and a uniformly distributed load w , the moment diagram at the design ultimate load level is shown in Fig 1.

In this case we have for the static moment at the design ultimate load level:

$$M_0^* = \frac{w^* L^2}{8} \quad (1)$$

The value of the negative design ultimate moment in the section at the support is:

$$\begin{aligned} |M_-^*| &= (1 + \beta) \frac{2}{3} M_0^* \\ &= (1 + \beta) \frac{2 w^* L^2}{3 \cdot 8} \end{aligned} \quad (2)$$

The positive mid-span design ultimate moment is:

$$\begin{aligned} M_+^* &= M_0^* - |M_-^*| \\ &= (1 - 2\beta) \frac{1}{3} M_0^* \\ &= (1 - 2\beta) \frac{1}{3} \frac{w^* L^2}{8} \end{aligned} \quad (3)$$

In these expressions a positive value of β means an increase in the value of M_-^* .

If the collapse load of such a beam, designed in accordance with AS 3600 to carry a design ultimate load w^* , is now determined by non-linear collapse analysis, and is $w_{u.rig}$, a system safety coefficient may be defined as:

$$\phi_{sys} = \frac{w^*}{w_{u.rig}} \quad (4)$$

With ϕ_{sys} evaluated in this way, a similar beam designed using non-linear collapse load analysis with the criterion

$$w^* \leq \phi_{sys} w_{u.rig} \quad (5)$$

would have a similar margin of safety to that provided by the ultimate section strength method of AS 3600. It is assumed here that the same load coefficients and load combinations are to be used in both the non-linear and the linear designs. For this reason the term ‘back-calibration’ is used here to describe this procedure for evaluating the system safety coefficient.

By considering a range of structural systems with values of the design parameters throughout the practical ranges it is possible to observe the variations in ϕ_{sys} which are required to achieve back-calibration to the currently used section strength limit states design methods.

Eibl(1991) has pointed out good reasons for using mean values of material properties (particularly strengths) in non-linear analysis and design. If direct non-linear design is to be based on mean strength properties of the steel and concrete, then clearly these values need to be used in the calculation of $w_{u.rig}$ and hence ϕ_{sys} , while the fractile values are used in choosing the design details of the members, according to the current standard and its section strength design procedure.

3. SYSTEM SAFETY COEFFICIENT FOR BEAMS WITH LIMITED DUCTILITY

The back calibration method works well provided the structures chosen for analysis satisfy the existing design requirements (in this case, AS 3600). However, one of the reasons for moving to the non-linear collapse method of design is its generality. It is potentially applicable to a wide range of structural systems without the restrictions imposed by current design concepts. A problem thus arises in how to obtain appropriate system safety coefficients for structural systems which fall outside of the existing standard. In the present case, we wish to evaluate system safety coefficients not only for ductile continuous beams, but also for beams with limited ductility.

According to AS 3600, structures can be designed with moment redistribution, provided the sections with peak moments have high ductility. Ductility is measured by the neutral axis parameter k_u calculated for ultimate moment M_u . In the case of the beam with fixed ends shown in Fig 1, the parameter β can be used as a measure of the design moment redistribution, where

$$\beta = \frac{0.5|M_{-}^{*}| - M_{+}^{*}}{M_0^{*}} \quad (6)$$

When $M_{+}^{*} = 0.5 / M_{-}^{*}$, the moment distribution is that obtained by linear elastic analysis, and $\beta=0$. Eq 6 above is obtained by solving simultaneously Eqs 2 and 3.

AS 3600 places limitations on β , depending on the values of k_u at the supports and at mid-span:

$$\text{If } \kappa_{v-} \leq 0.2 \text{ and } \kappa_{v+} \leq 0.2 : \quad -0.3 \leq \beta \leq +0.3$$

$$\text{If } \kappa_{v-} \geq 0.4 \text{ and } \kappa_{v+} \geq 0.4 : \quad \beta = 0.0$$

$$\text{For various intermediate values: } -1.5 (0.4 - \kappa_{vmax}) \leq \beta \leq +1.5(0.4 - k_{umax}), \text{ where } k_{umax} \text{ is the larger of } k_{u-} \text{ and } k_{u+}.$$

In reality, of course, these are safe, conservative design provisions. Moment redistribution does not cease to occur when the k_u values marginally exceed 0.4. The result, however, is that the AS 3600 design can become extremely conservative when substantial moment redistribution is ignored. This in turn leads to a highly conservative estimate of w^* and hence to an underestimate of ϕ_{sys} from Eq 4. To obtain reasonable values for ϕ_{sys} for beams with limited ductility, some form of extrapolation is needed. In the present study, the load capacity $w_{u.rig}$ is calculated for a given beam together with the actual moments at the supports and at mid-span when the load capacity is reached, i.e. $M_{-.rig}$ and $M_{+.rig}$. These are not necessarily the moment capacities of the sections. The amount of moment redistribution at $w_{u.rig}$ can be evaluated as:

$$\beta_{u.rig} = \frac{0.5 |M_{-.rig}| - M_{+.rig}}{M_{0.rig}} \quad (7)$$

where

$$M_{0.rig} = \frac{w_{u.rig} L^2}{8} \quad (8)$$

and it is reasonable to assume that this is the moment redistribution which can be used in design, which will be otherwise in accordance with AS 3600. Using $\beta_{u.rig}$ instead of β for design allows a reasonable value of w^* to be obtained. Rearranging Eq 2 and replacing β with $\beta_{u.rig}$ gives:

$$w^* = \frac{8}{L^2} \frac{3}{2} \frac{|M_-^*|}{(1 + \beta_{u.rig})} \quad (9)$$

This value has been used in Eq 4 to evaluate ϕ_{sys} for those beams with limited ductility, i.e. those which fall outside of the design requirement of AS 3600. For comparison purposes, w^* has also been calculated using $\beta_{u.rig}$ for beams which met the design requirement of AS 3600.

4. DESIGN AND ANALYSIS OF FLEXURAL MEMBERS

In order to obtain values of ϕ_{sys} using the back-calibration method, a large number of beams were designed and then analysed using a fully non-linear method of collapse analysis, which has previously been described by Wong and Warner(1997).

All of the beams were single span, with either one or both end fixed, as shown in Fig 1. The beams with fixed ends represent, approximately, the conditions in interior spans of continuous beams, while the propped cantilevers approximate those in end spans. The cross-section in all beams was rectangular, 400mm wide and 800mm deep, with a cover of 50mm to the centroid of both the top steel and bottom steel. All calculations were carried out assuming load coefficients of 1.25 and 1.5 for dead and live load, and a dead/live load ratio of unity. The design data used in the study are:

- 1) concrete characteristic strength at 28 days $f_c' = 32$ MPa, with mean strength at 28 days $f_{cm} = 1.085 f_c' + 2.5 = 37.2$ MPa
- 2) reinforcing steel yield strength $f_{sy} = 400$ MPa , with mean strength $f_{sm} = 460$ MPa
- 3) two load patterns: point load at mid-span, uniformly distributed load
- 4) span to depth ratios of 10, 20, 30
- 5) k_u of the region next to the support of 0.1, 0.2, 0.3, 0.4, 0.4a
- 6) tensile reinforcement in the positive bending moment region ranging from 1000 mm² to 12 000 mm²
- 7) different support conditions: simply supported, one end fixed, both ends fixed

The formulae for the mean concrete strength f_{cm} above is from C&CA records (see Prestressed Concrete Design Consultants Pty Ltd(1975)). Table 1 shows that the values of mean concrete strength obtained using this formulae agree well with those given in the Concrete Design Handbook (C&CAA and SAA, 1995).

The designation of k_u of 0.4a is for sections with the area of steel in tension A_{st} of $0.04 bd$, where b is the width and d is the effective depth, and with the amount of steel in compression A_{sc} determined base on the section having a k_u value of 0.4. In comparison, the designation of $k_u = 0.4$ refers to singly reinforced sections at supports with k_u just reaching 0.4. In the positive bending moment regions of the beams, where k_u exceeds 0.4 if singly reinforced, appropriate amount of compression steel was included to reduce k_u value to 0.4. This was to comply with the AS 3600 requirement that k_u value for flexural members is not to exceed 0.4.

The following mean values of material properties were assumed for the non-linear analysis: young modulus for steel, $E_{sm} = 2.0E+05$ MPa; young modulus for concrete, $E_{cm} = 5050\sqrt{f_{cm}}$ MPa; parameter γ_2 of 3.0 was used in the curvilinear stress-strain relation (Warner, 1969). The concrete peak stress was assumed to be equal to f_{cm} , and the strain at this peak stress was assumed to be 0.002. Steel reinforcement was assumed to have an elastic-plastic stress-strain relation. Tension-stiffening effect was included by using the stress-strain relation for concrete in tension proposed by Kenyon and Warner(1993).

Table 1: Mean concrete strength

Characteristic strength, f_c'	Mean concrete strength at 28 days, f_{cm}	
	Concrete Design Handbook	Eqn from C & CA records
20	24.0	24.2
25	29.5	29.6
32	37.5	37.2
40	46.0	45.9
50	56.5	56.8

5. FAILURE MODES OF FLEXURAL MEMBERS

Load versus mid-span deflection curves for various fixed-ended beams with a span to depth ratio of 30, subjected to a uniformly distributed load w , and with a k_u at the supports of 0.2 are shown in Fig 2. The twelve curves are for A_{st} values at mid-span ranging from 1000 mm² to 12000 mm². The beam with the lowest ultimate strength has the following reinforcement in the positive bending moment region: $A_{st} = 1000$ mm² and $A_{sc} = 0$; the beam with the highest ultimate strength has the following reinforcement: $A_{st} = 12000$ mm² and $A_{sc} = 5290$ mm².

Results from the non-linear analysis show that for beams with ductile behaviour, the ultimate load remains quite constant after the reinforcing steels in both the positive and the negative bending moment regions have yielded; this remains so until softening commences in the negative bending moment regions. Beams with a large amount of reinforcement in the positive bending moment region may reach ultimate load before the onset of yielding of the steel in the positive bending moment region. For these beams, softening of the negative bending moment regions occur before the onset of steel yielding in the positive bending moment region. This behaviour is consistent with earlier observations by Darvall(1983) that load softening can be induced by severe softening in only one hinge.

For those beams with A_{st} at mid-span ranging from 1000 mm² to 4000 mm², their load versus deflection curves each shows the presence of a peak-load plateau, i.e. a region with quite constant load-deflection relation.

This shows that the steel reinforcement at mid-span has yielded before the beam reaches its ultimate load. In contrast, those with A_{st} at mid-span of at least 5000 mm² do not show any plateau in their load versus deflection curves. The absence of a plateau indicates that steel yielding has not yet occurred at mid-span when collapse occurs. These beams have insufficient ductility at their support sections to facilitate the formation of a plastic collapse mechanism; their ultimate loads were reached before the onset of steel yielding at mid-span, and was the result of limited ductility in the negative moment regions.

6. SYSTEM SAFETY COEFFICIENTS FOR BEAMS SATISFYING AS 3600 REQUIREMENTS

Of the 1404 beams studied, 251 satisfied the requirements of AS 3600. These requirements are:

- total deflection limit of $L/250$ at working load; and
- moment redistribution and ductility limits.

The deflection at working load was determined from the non-linear analysis, and this is described in Appendix A. The ratio of dead load to live load was assumed to be unity.

The system safety coefficients ϕ_{sys} for these 251 beams were determined and their rounded values are presented in the histogram given in Fig 3. The values of ϕ_{sys} were obtained from w^* calculated with the moment redistribution factor in accordance with AS 3600. For these members, values of ϕ_{sys} were also determined using the moment redistribution factor actually achieved, $\beta_{u.rig}$, in the non-linear analysis. These values are presented in the histogram given in Fig 4. In Fig 3 the value of ϕ_{sys} lies mainly between 0.67 and 0.69, and in Fig 4 mainly between 0.68 and 0.70. The differences are small.

This first back-calibration exercise suggests that for beams which satisfy AS 3600 design requirements, a system safety coefficient ϕ_{sys} of 0.68 may be used for non-linear design.

7. SYSTEM SAFETY COEFFICIENTS FOR DUCTILE BEAMS

The rounded values of system safety coefficient based on β and $\beta_{u.rig}$ for 1154 beams with sufficient ductility to ensure yielding in both the positive and negative bending moment regions are represented by the histograms shown in Figs 5 and 6, respectively. Some of these beams do not satisfy the AS 3600 requirements for ductility.

Again these figures show that a system safety coefficient ϕ_{sys} of 0.68 may be used for non-linear design. Similar to the results shown in Fig 3, Fig 5 shows that the value of ϕ_{sys} lies mainly between 0.67 and 0.69, and similar to Fig 4, Fig 6 shows that the value of ϕ_{sys} lies mainly between 0.68 and 0.70. These figures suggest that those beams which do not satisfy AS 3600 requirements, owing to either excessive amount of moment redistribution at collapse or excessive deflection at working

load, but with sufficient ductility to give a plastic collapse mechanism at collapse, give system safety coefficients comparable to those which satisfy AS 3600 requirements.

A non-linear design calculation was also carried out for these beams using a system safety coefficient value of 0.68. The ultimate design load was obtained from the following expression below (based on Eq 5):

$$\begin{aligned} w^*(\text{non-linear}) &= \phi_{\text{sys}} w_{u.\text{rig}} \\ &= 0.68 w_{u.\text{rig}} \end{aligned} \quad (10)$$

A histogram of the ratio $w^*(\text{non-linear})$ to w^* , where w^* is the ultimate design loads obtained using the AS 3600 section-strength design method, is shown in Fig 7. This figure also supports the use of a system safety coefficient of 0.68.

8. GLOBAL SAFETY COEFFICIENT FOR NON-DUCTILE BEAMS

The rounded values of system safety coefficient based on $\beta_{u.\text{rig}}$ for 250 beams with insufficient ductility to form a plastic collapse mechanism at failure are represented by the histogram shown in Fig 8. As explained in Section 3, β cannot be obtained for these beams owing to their limited ductility.

These beams, generally, give system safety coefficients larger than those obtained for the ductile beams described in Section 7. However, care must be taken when using such beams in design as their collapse mode, characterised by the non-formation of a collapse mechanism, may give insufficient warning of impending collapse. Such behaviour is highly undesirable, and is to be avoided. Use of the system safety coefficient value of 0.68 seems to be appropriate for these beams because it is on the conservative side.

9. MEANS AND STANDARD DEVIATIONS OF GLOBAL SAFETY COEFFICIENT

The mean and standard deviation for the system safety coefficients for the beams described above are summarised in Tables 2 and 3. The mean

and standard deviation values for beams which satisfy AS 3600 requirements based on β are 0.681 and 0.008, respectively; those based on $\beta_{u.rig}$ are 0.688 and 0.009, respectively. The mean and standard deviation values of system safety coefficient for beams with plastic collapse mechanism based on β are 0.682 and 0.008, respectively; those based on $\beta_{u.rig}$ are 0.690 and 0.010, respectively. The mean values are close to the value of 0.68 proposed earlier for use with non-linear design.

Table 2: ϕ_{sys} for beams (using β)

Description	ϕ_{sys}	
	Mean	Std dev.
(1) Beams satisfying AS 3600 (251 members)	0.681	0.008
(2) Beams with plastic collapse mechanism (1154 mem.)	0.682	0.008

Table 3: ϕ_{sys} for beams (using $\beta_{u.rig}$)

Description	ϕ_{sys}	
	Mean	Std dev.
(1) Beams satisfying AS 3600 (251 members)	0.688	0.009
(2) Beams with plastic collapse mechanism (1154 mem.)	0.690	0.010

10. EFFECT OF SUPPORT CONDITION ON SYSTEM SAFETY COEFFICIENT

In this section, the effect of the support conditions on ϕ_{sys} is investigated. Fig 9 shows the histogram for system safety coefficient based on β for beams with different support conditions which satisfy the AS 3600 design requirements. The abbreviations used in the legend are: fix0 = simply supported, fix1 = propped cantilever, and fix2 = fixed ended.

Each of the support conditions gives ϕ_{sys} mainly within the range of 0.67 to 0.69. Some simply supported beams give system safety coefficient as low as 0.64, and some fixed-ended beams give system safety coefficient as high as 0.70. There is a very slight tendency for the system safety coefficient to be more conservative for the beams with both ends fixed and to be less conservative for the simply supported beams. However,

the lack of trend suggests that the support condition has little effect on the system safety coefficient.

The histogram for rounded system safety coefficient based on β for the 1154 beams with different support conditions and which form a mechanism at collapse is shown in Fig 10. Some beams with no fixed support, i.e. simply supported, give a system safety coefficient value as low as 0.64. Here, as in the case when considering only those beams which satisfy AS 3600 requirements, there is no observable trend for the effect of support condition on ϕ_{sys} other than a slight tendency for the simply supported beams to take on lower ϕ_{sys} values.

The mean and standard deviations of ϕ_{sys} for beams with different support conditions are given in Table 4.

The mean value for the three different support conditions ranges from 0.67 to 0.70. There is no general trend in the relationship between the value of ϕ_{sys} with the degree of fixity, which again suggests that support condition has little effect on system safety coefficient.

Table 4: ϕ_{sys} values for beams with different support conditions

Support type	Value of ϕ_{sys}								
	members to AS 3600 (based on β)			members to AS 3600 (based on $\beta_{u.rig}$)			All members (based on $\beta_{u.rig}$)		
	total no.	mean	std dev	total no.	mean	std dev	total no.	mean	std dev
fix0	21	0.673	0.018	21	0.673	0.018	71	0.672	0.014
fix1	85	0.680	0.007	85	0.692	0.005	651	0.697	0.012
fix2	145	0.684	0.004	145	0.690	0.006	682	0.694	0.010

11. EFFECT OF CONCRETE STRENGTH ON SYSTEM SAFETY COEFFICIENT

To investigate the effect of concrete strength on the system safety coefficient, a second series of beam calculations were carried out using the following:

- 1) concrete characteristics strength grades $f_c' = 25, 32, 40$ and 50 MPa, with $f_{cm} = 1.085 f_c' + 2.5$

- 2) yield strength of reinforcing steel $f_{sy} = 400$ MPa , with corresponding $f_{sm} = 460$ MPa
- 3) one load pattern : uniformly distributed load
- 4) span to depth ratio of 20
- 5) k_u of the region next to the support of 0.1, 0.2, 0.3, 0.4, 0.4a
- 6) tensile reinforcement in the positive bending moment region ranges from 1000 mm^2 to 12000 mm^2
- 7) support condition : both ends fixed

The mean and standard deviation for system safety values obtained from this study are given in Table 5. This table shows that for normal strength concrete, the system safety coefficient is quite insensitive to the strength grade of concrete.

Table 5: ϕ_{sys} values for beams with different characteristic concrete strengths

f'_c	ϕ_{sys}								
	members to AS 3600 (based on β)			members to AS 3600 (based on $\beta_{u,rig}$)			All members (based on $\beta_{u,rig}$)		
	Mean	Std dev.	Total no.	Mean	Std dev.	Total no.	Mean	Std dev.	Total no.
25	0.682	0.005	16	0.687	0.007	16	0.701	0.012	112
32	0.685	0.003	16	0.688	0.006	16	0.700	0.011	107
40	0.686	0.004	31	0.692	0.005	31	0.699	0.010	118
50	0.687	0.005	36	0.694	0.005	36	0.698	0.009	122

12. CONCLUSIONS

The back-calibration procedure used in this extensive numerical study suggests that a system safety coefficient of 0.68 may be used for non-linear design of reinforced concrete flexural members. This applies to beams with steel reinforcement with a yield strength of 400 MPa and a corresponding mean strength of 460 MPa. This value is applicable to both ductile beams and beams with limited ductility.

The system safety coefficient for non-linear design has been found to be quite insensitive to the degree of fixity at the supports of the structures and the strength of the concrete used in the structures.

13. ACKNOWLEDGMENT

The work described in this report is funded by a large ARC (Australian Research Council) grant.

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APPENDIX A: TOTAL DEFLECTION CALCULATION TO AS 3600

The total deflection of a beam at mid-span is calculated using Eq A1.

$$\Delta_{tot} = \Delta_s + k_{cs} \Delta_{s,sus} \quad (A1)$$

From the non-linear load deflection relation obtained from the non-linear analysis, determine Δ_s which corresponds to the short-term load $w_g + \psi_s w_q$. Also from the non-linear analysis determine $\Delta_{s,sus}$ which corresponds to the sustained load $w_g + \psi_l w_q$. Linear interpolation is used to obtain values if they fall between two solution points. Here w_g is the dead load, w_q is the live load, ψ_s is the short-term load coefficient for serviceability design and ψ_l is the long-term load coefficient for serviceability design. The long term deflection multiplier, $k_{cs} = [2 - 1.2(A_{sc}/A_{st})]$, and is to be greater or equal to 0.8.

For the present study, the following assumptions were made:

- 1) $w_g = w_q = w^*/2.75$, from solving simultaneously equations:
 $1.5 w_g + 1.25 w_q = w^*$ and $w_g = w_q$
- 2) Values of $\psi_s = 0.7$ and $\psi_l = 0.4$ based on the type of live load being that for offices (Standard Australia, 1989)

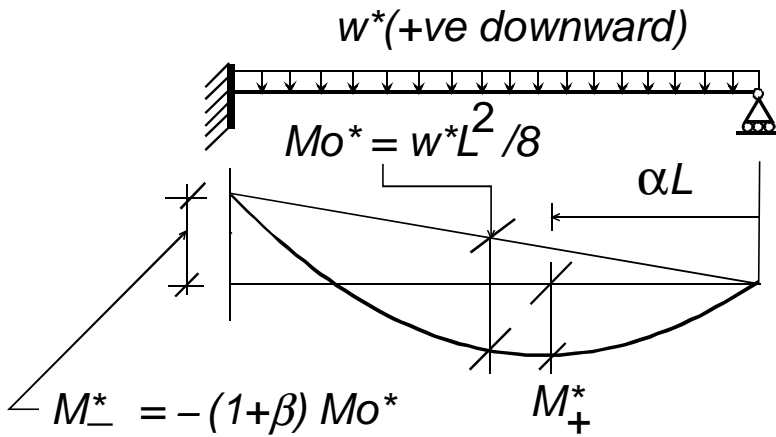
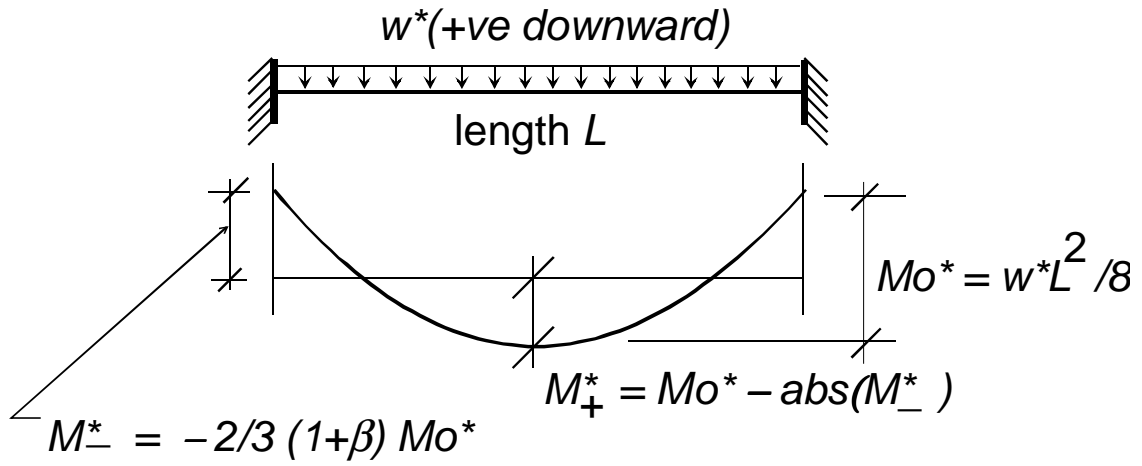


Figure 1: Bending moment diagrams

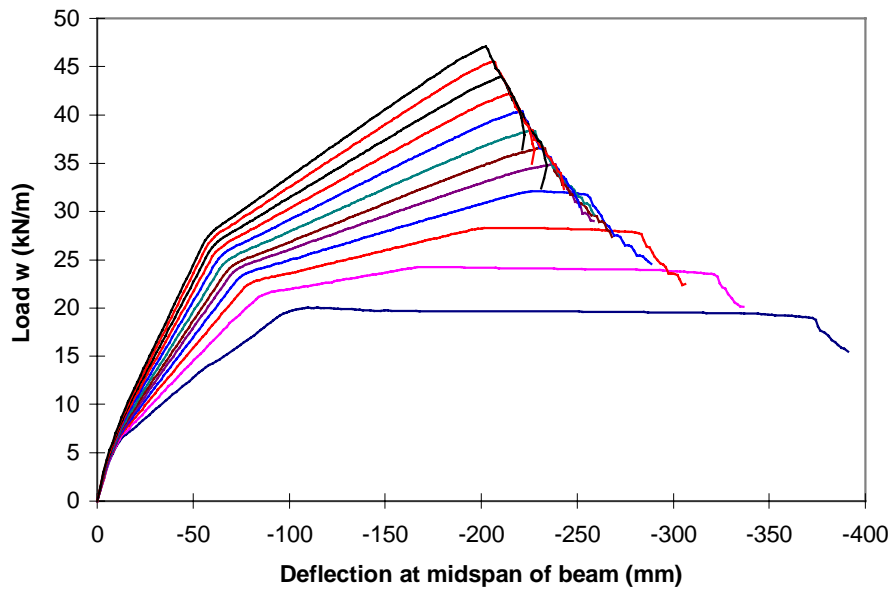


Figure 2: Load versus deflection plots for fixed-ended beams

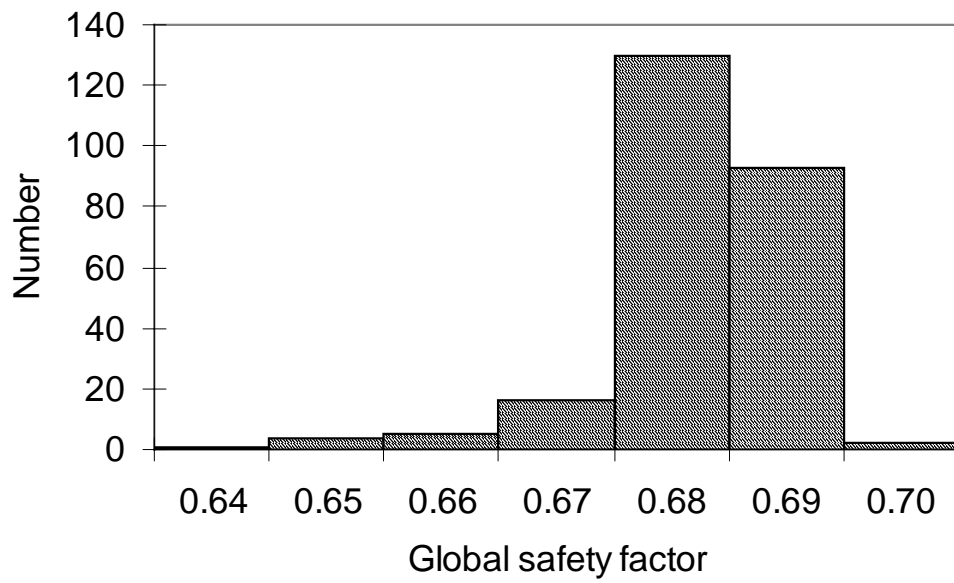


Figure 3: System safety coefficient for the 251 members which satisfy AS 3600 requirements (using β)

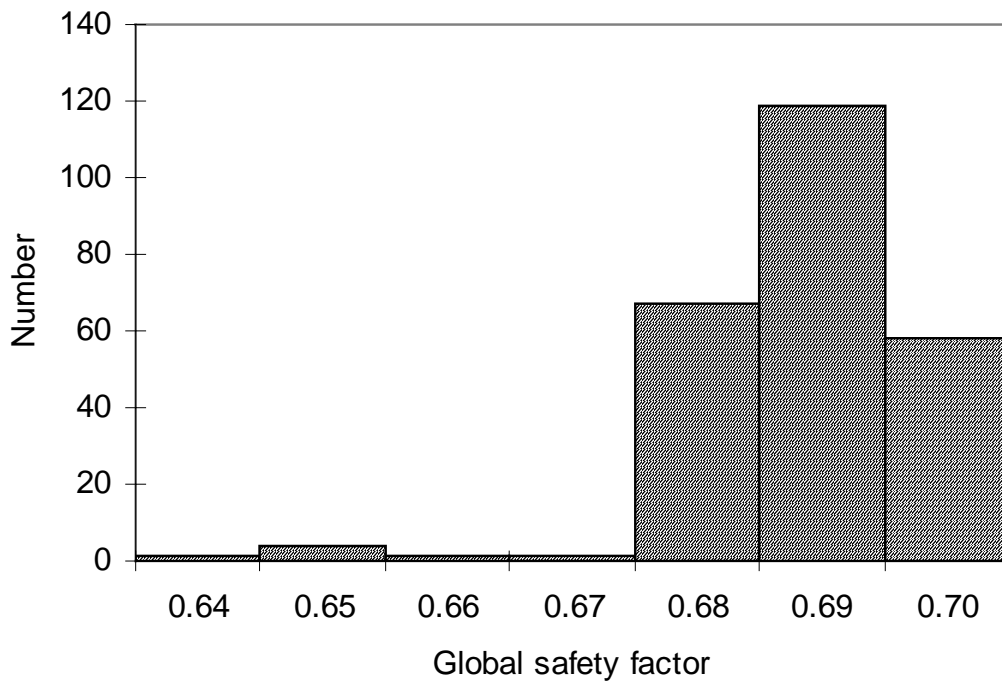


Figure 4: System safety coefficient for the 251 members which satisfy AS 3600 requirements (using $\beta_{u,rig}$)

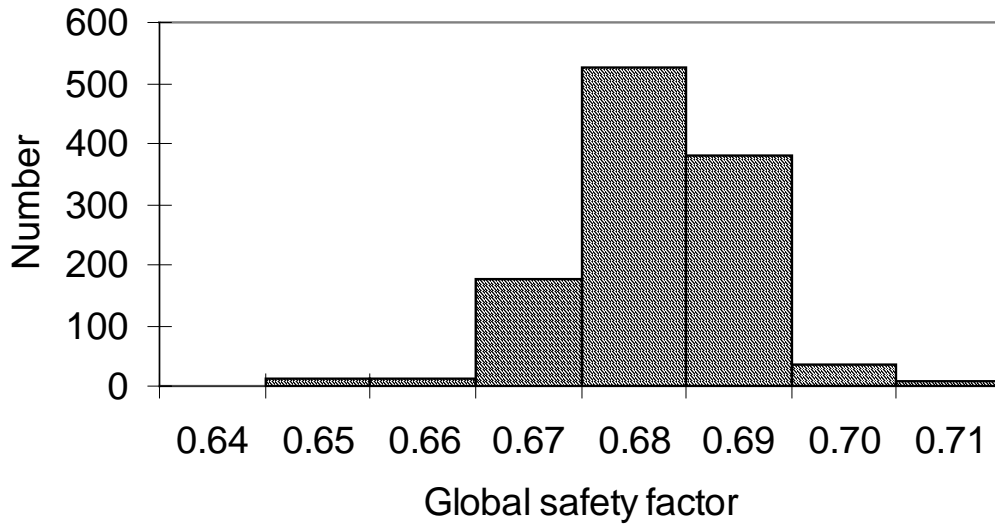


Figure 5: System safety coefficients for 1154 beams with plastic mechanism at collapse (using β)

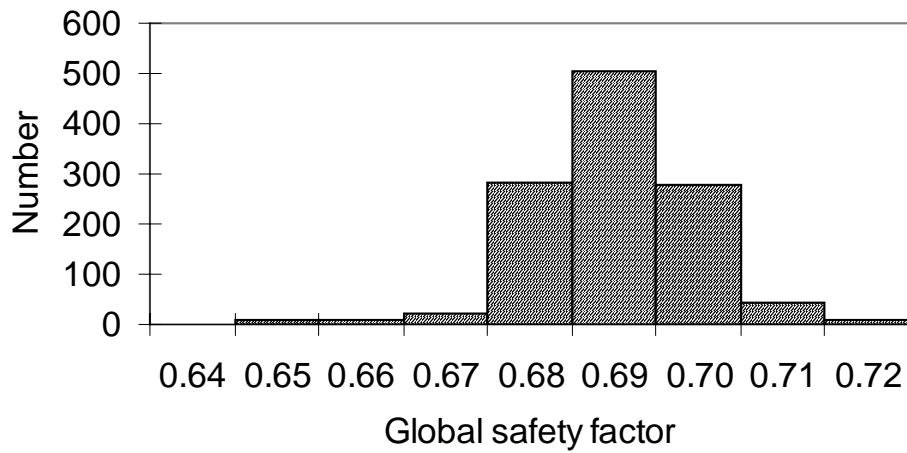


Figure 6: System safety coefficients for 1154 beams with plastic mechanism at collapse (using $\beta_{u.rig}$)

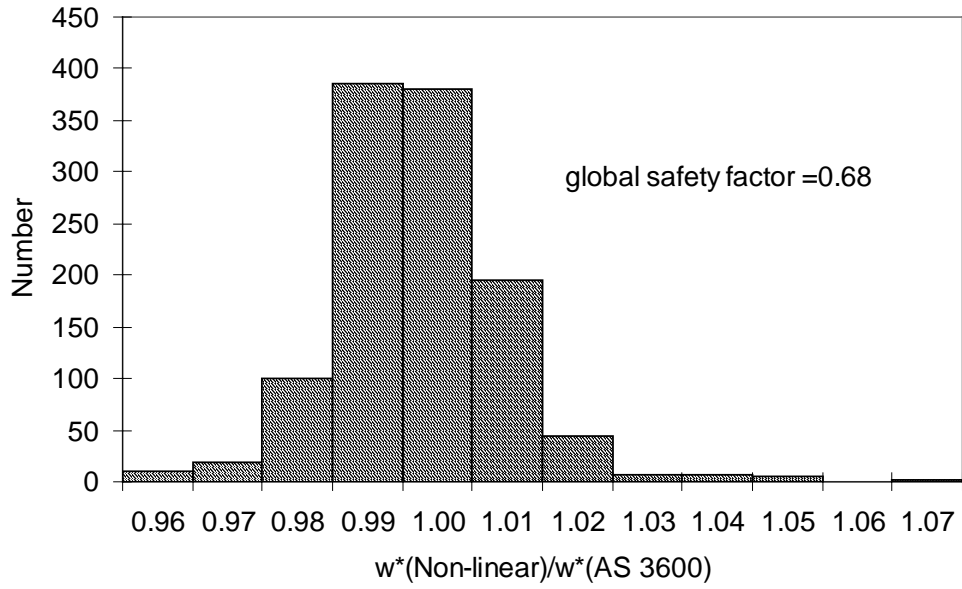


Figure 7: Ratio of w^* (non-linear) to w^* for 1154 beams with plastic mechanism at collapse (using $\phi_{sys} = 0.68$)

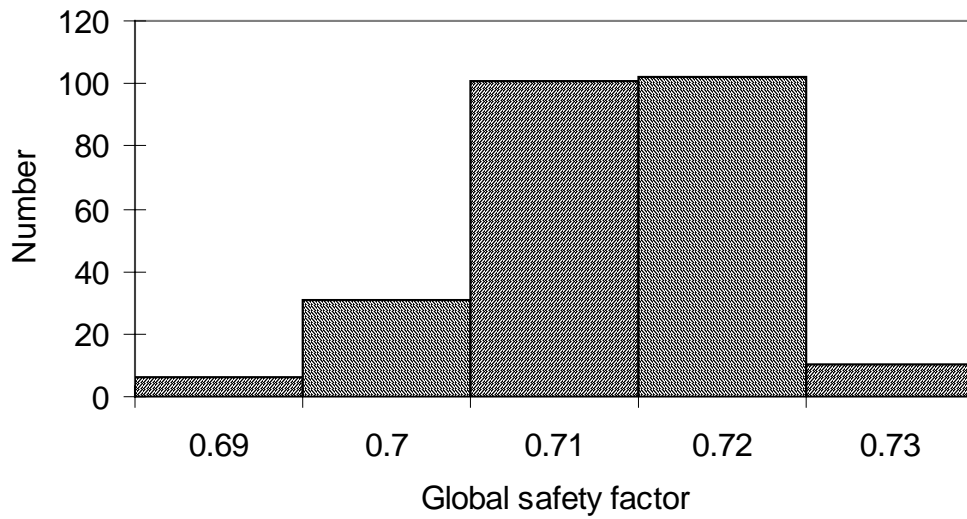


Figure 8: System safety coefficients for 250 beams with limited ductility (using $\beta_{u.rig}$)

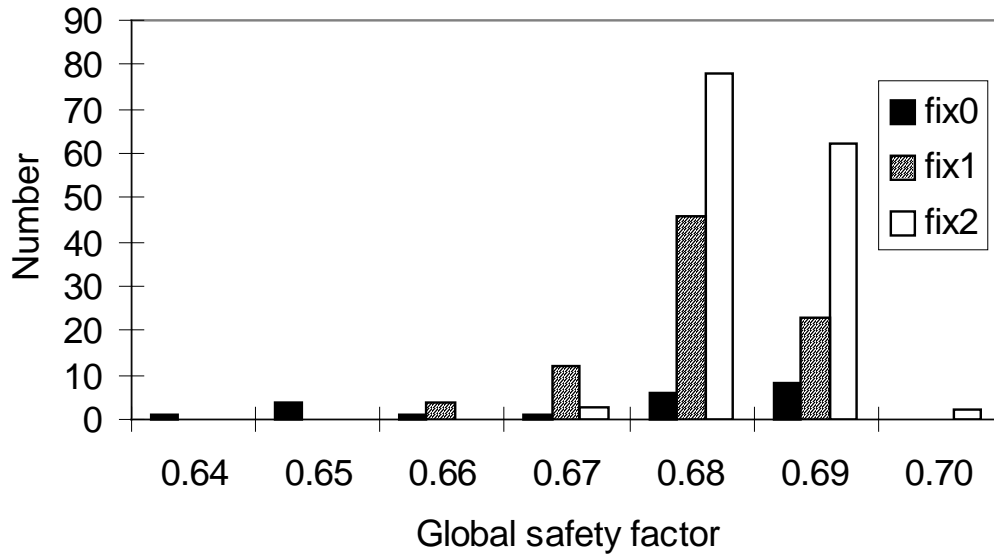


Figure 9: ϕ_{sys} for 251 beams which satisfy AS 3600 requirements (using β)

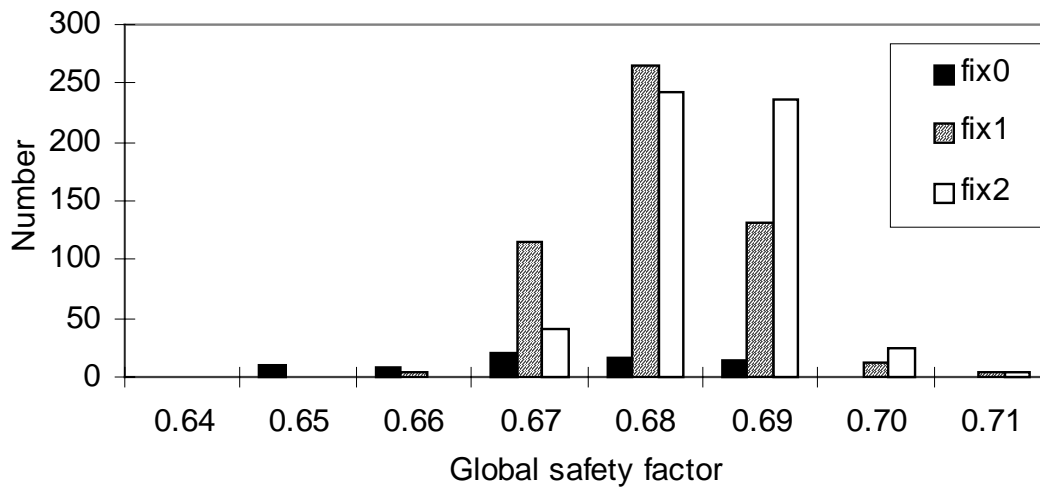


Figure 10: ϕ_{sys} for 1154 beams with plastic mechanism at collapse (using β)