

Collapse Load Analysis of Prestressed Concrete Structures

by

K W Wong

R F Warner

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K W Wong
R F Warner

Department of Civil and Environmental Engineering
The University of Adelaide

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K W Wong
Research Fellow

R F Warner
Professor

Department of Civil and Environmental Engineering
University of Adelaide

ABSTRACT: This report describes a non-linear method of analysis for prestressed concrete structures which predicts behaviour at all stages of loading, from the initial application of prestress up to and beyond the collapse condition. The method uses a segmental line element approach and is a modification of an existing method previously developed for reinforced concrete structures. Comparisons with the results of tests of continuous prestressed concrete beams show good correlation. The method has been developed for the purpose of investigating the collapse behaviour of indeterminate prestressed concrete structures and hence to develop design recommendations on minimum acceptable ductility levels, moment redistribution and design safety coefficients.

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1. INTRODUCTION

This report describes a non-linear method of analysis for prestressed concrete structures at working load and at collapse.

The method of analysis was developed as a tool for investigating non-ductile behaviour in prestressed concrete flexural structures and hence for the development of guidelines on minimum ductility requirements for the design of prestressed concrete structures.

The accuracy of the analytic procedure has been evaluated using previously published test data for continuous prestressed beams. The predicted behaviour was found to be in reasonably good agreement with the test results.

The analytic method is currently being used in parametric studies of the factors which affect local section ductility and the modes of collapse of prestressed concrete continuous beams. Of particular interest in these studies is collapse by premature fracture of the prestressing tendon. Parametric studies are also being carried out to evaluate global safety coefficients for use in the non-linear design of prestressed concrete structures.

2. NON-LINEAR ANALYSIS OF PRESTRESSED CONCRETE STRUCTURES

Various methods of analysis have been used for indeterminate prestressed concrete structures. These include the methods developed by Kang and Scordelis (1980), Warner and Yeo (1984); Kgoboko (1987); Kgoboko, Wyche and Warner (1988); and Campbell and Kodur (1990).

Kang and Scordelis (1980) described a numerical procedure based on a finite element formulation for non-linear analysis of plane prestressed concrete frames. The method uses a tangent stiffness formulation to calculate incremental displacements. These displacements are added to the latest total displacements, and a secant stiffness formulation is then used to determine the resisting loads at the joints based on the latest state of materials and geometry of the structure. From this and the latest applied loads, the unbalanced joint forces are calculated for use in the next iterative cycle. Several such iterative cycles are required to achieve convergence. This is usually known as the predictor-corrector method,

the predictor uses the tangent stiffness formulation, and the corrector uses the secant stiffness formulation. This non-linear analysis is carried out by incrementing load, and is not therefore suitable, without modification, for tracing the collapse behaviour of concrete structures up to and beyond collapse.

Warner and Yeo (1984) described a line element direct stiffness approach to study partially prestressed concrete beams. In this approach, a structure is modelled by dividing members into segments, each represented by a single line element. A curvature control procedure, first introduced by Warner (1984), is used to trace the full-range behaviour of the structure. The curvature increment procedure has been found to be superior to procedures based on incrementing loads for tracing the full-range behaviour of non-linear structures. Moment curvature relations for segments were pre-generated, and these relations were used in the structural analysis.

Kgoboko (1987) used the same approach as Warner and Yeo in his study of the collapse behaviour of partially prestressed concrete structures but improved the efficiency by using segmented elements to represent structural members. Segmented elements were first used by Wong et al (1987) in a study on the collapse behaviour of reinforced concrete structures. With this approach, a beam can be modelled as a single segmented element. This reduces greatly the size of the structural stiffness matrix by reducing the number of nodes, which in turn reduces both computer storage and program execution time. A linear-elastic analysis was carried out to determine the initial effect of prestressing and self-weight. This approach was used to study the ductility of prestressed concrete bridge girders (Kgoboko, Wyche and Warner, 1988).

Campbell and Kodur (1990) also used pre-generated moment-curvature relations and a curvature-increment solution procedure for non-linear analysis of prestressed concrete continuous beams. This approach is similar to that of Warner and Yeo, but different from that of Kgoboko, as non-segmented elements were used to represent a member. Campbell and Kodur also used a linear elastic analysis to determine the effect of prestressing.

3. PRESENT METHOD OF ANALYSIS

The method of analysis described here for prestressed concrete structures has been developed by adapting an existing procedure for reinforced concrete structures. This method was chosen for adaptation because it has been in use for many years and has proven to be numerically stable, robust and computationally efficient. The method also can take full account of geometric and material non-linearities.

Although the method has been described previously (Wong et al, 1987, 1988), some of the key features will be mentioned briefly here because of their relevance to the analysis of prestressed concrete structures.

Each member in the structural system is discretised into a number of line elements and each of these is further discretised into a number of segments. The flexural behaviour of each segment is determined at any stage of loading on the structure by using a section analysis in which a typical cross-section is represented as a large number of thin layers of concrete and reinforcing steel. For a given or assumed strain distribution in the section, stress-strain relations for the component materials are used to determine stresses and hence forces in the various layers. Summation of the layer forces and their moments give the axial force and moment acting in the section. In situations where loading and unloading occur in a section, the strain history of each layer is used to determine the current stress and hence the stress resultants.

In this way, fully non-linear material behaviour can be considered, including cracking of the concrete in tension, tension stiffening, compressive softening of the concrete, and yielding, strain hardening or unloading of reinforcing steel.

For the analysis of the structural system, a secant modulus method is adopted. The secant stiffness of each segment in an element is determined on a trial basis and used to establish a trial secant modulus for the element. The secant stiffnesses of the segments in an element are used to form the element stiffness matrix of the segmented element. The terms in the element stiffness matrix are given in Appendix A. The element stiffnesses are then used to form the trial system stiffness at the particular stage of loading being considered.

Loading along the segmented element is included by using equivalent nodal forces based on fixed end moments. Equations for fixed-end moments for a segmented element are given in Appendix B.

The structural analysis is carried out by defining a “unit” pattern load. A key segment is chosen, and increments of curvature are applied to this segment. The load factor λ is then determined for each incremental curvature in the key segment until collapse occurs.

3.1 Modifications for Treating Prestressed Concrete

In order to expand the existing analysis method to treat prestressed concrete structures, two main modifications were required.

Firstly, a preliminary analysis was added to take account of the introduction of the prestressing force into the concrete-steel structure. In this analysis the self-weight of the structure is considered to be introduced at the same time that the prestress is applied. While this usually occurs in practice, it is also convenient to treat these processes simultaneously for computational reasons. This is discussed further in Section 3.2 below.

The second main modification made to the main structural analysis was to take account of the presence of the prestressing steel when determining the stiffness and section stresses in local elements during the post-cracking and overload stages of behaviour.

These two modifications are now considered in turn.

3.2 Stage 1: Application of Prestress and Self-weight

In this preliminary analysis, the application of the prestressing force to each concrete element is considered. Initially, the case of post-tensioned elements which contain a curved prestressing cable is considered. As shown in Fig 1a, the curved prestressed cable applies a non-uniformly distributed force to the concrete element.

To approximate this situation in the analysis, the cable is considered to be straight within each segment, and to have a slight kink at each juncture between adjacent segments (Fig 1b). Statically, the distributed load acting on the concrete in Fig 1a is being represented approximately by a series of

small inclined discrete forces acting at the junctures between elements. The real cable at any point is typically eccentric to the centroidal axis and inclined and curved. This produces a uniformly distributed force on the concrete segment under consideration which is perpendicular to the cable. The force applied at each juncture thus has a vertical and a horizontal component. Furthermore, to allow for the eccentricity of the real cable, a moment also acts at the juncture, as shown in Fig 2c. The system of vertical and horizontal forces and moments acting at each juncture along the element is statically equivalent to the continuous force system being applied to the element by the prestressed cable

In most practical situations, the slope of the cable is relatively small and only the vertical component is of significance. Expressions for vertical forces from the prestressed cable at the junctures between segments and at the nodes of a segmented element are given in Appendix C. This means that both the horizontal force and the moment (equal to the horizontal force times the cable eccentricity at the node) are very small. While both the horizontal force and the moment can easily be allowed for, we ignore them in the following analysis.

If an analysis is undertaken of a practical structure with only the prestress acting, it is often found that cracking of the concrete is predicted. In the simple beam in Fig 1 for example, the prestressing produces large tensile stresses in the upper fibres. This is because the cable is designed to partially balance the stresses due to external load. It is convenient therefore to analyse the initial state of the structure with the effects of both the prestress and self-weight. If these effects are analysed separately, spurious non-linear effects are introduced because of cracking. Because the behaviour in the post cracking stage is significantly non-linear, it is not possible to treat the two effects separately and superpose the results.

The continuous body forces due to self-weight are represented by a uniformly distributed force acting along the element.

The structure is analysed for the simultaneous action of the forces due to prestress and self-weight. A trial procedure is used which usually requires only one or two iterations. In the first cycle all concrete layers throughout the structure are assumed to be uncracked, and the secant modulus of elasticity of each layer of material is set equal to the initial tangent modulus for the material. The stiffness of each segment, and hence of each element and finally the system, is determined and a linear analysis is undertaken.

The Stage 1 analysis is illustrated by the flow diagram shown in Fig 3.

The steps in the analysis are:

1. Determine the equivalent loads acting on the system from the prestressing cable. These act at the junctures between segments and at the ends of the elements. The equivalent loads at the end of an element with tendon anchorage include a horizontal prestressing force acting along the reference axis, and a moment acting on the section at the reference axis if the tendon is anchored below or above the reference axis.
2. Determine the self-weight acting on the system. This is modelled by representing the self-weight of a horizontal element as an equivalent uniformly distributed load acting along it.
3. The total load pattern from equivalent loads and loads from self-weight is obtained by superimposing the two loads described above.
4. Divide each individual load (self-weight and equivalent prestress loads) by the magnitude of the effective prestressing cable force F_{prest} to obtain the unit load pattern; this unit load system is scaled by a load scaling factor λ . A load-increment procedure is used to analyse the system for progressively increasing load until the full load pattern with λ equal to F_{prest} is reached. Ten equal load increments are used, each of magnitude $F_{prest}/10.0$, to reach the final state of $\lambda = F_{prest}$. For each increment in load, the system under the latest load pattern is analysed taking into consideration material non-linearities.

The use of a load-increment procedure for this first stage of analysis is acceptable as the system is assumed not to be near collapse at the end of this stage of analysis.

3.3 Stage 2: Post-cracking Behaviour up to Collapse

In Stage 2, the system is subjected to dead and increasing live load. The procedure, at present limited to proportional loading, is illustrated by the flow diagram shown in Fig. 4.

The Stage 2 loading commences after the tendon has been bonded to the concrete. The stresses and strains of the concrete and non-prestressing steel at the end of Stage 1 are stored as reference stresses and strains.

The stress and strain in the prestressing tendon are obtained from the effective cable force and the cross-sectional area of the tendon. Therefore, the tensile strain in the bonded tendon is much higher than the tensile strain in the concrete surrounding it.

The steps in the Stage 2 analysis are:

1. Define the “unit” load pattern from the dead and live load. This unit load pattern is subjected to a load scaling factor λ , values of which are determined for increasing curvatures in a chosen key segment.
2. At the beginning of Stage 2 loading, the structural system is set to its reference configuration; that is, deflections and curvatures are all zeros. All action effects obtained from the Stage 2 analysis are relative to their reference values stored at the beginning of stage 2.
3. A curvature increment procedure is used to trace the non-linear Stage 2 behaviour. This is basically the same as the procedure for reinforced concrete structures described in earlier papers (Warner, 1984; Wong et al, 1988), but with the reinforcing steels (including the tendon) and the concrete having pre-strains and pre-stresses. Note that as the tendon is bonded to the concrete at the beginning of stage 2, the tendon and concrete have the same Stage 2 strain on loading but different total strains due to different starting strains as a result of the prestressing and the subsequent grouting of the prestressing tendon.

The main changes are in the section analysis. The Stage 2 curvature obtained during the system analysis is relative to the reference curvature. Therefore, for a typical material layer, Stage 2 strain is added to its reference strain to give the total strain. The total strain in turn gives the total stress based on the assumed material stress-strain relation, taking into consideration material unloading. After having obtained the total stress, the Stage 2 stress is obtained by subtracting the reference stress from the total stress. The flexural rigidity for the section (and also the segment) is obtained by dividing the Stage 2 bending moment (calculated using the Stage 2 stresses) by the Stage 2 curvature.

One simplifying assumption being made is that the cable is straight within a segment, and the eccentricity used is that at the mid-segment. The segment is assumed to be uniform with a characteristic EI value equal to that of the end section with the larger bending moment. This assumption is conservative in most situations.

4. COMPARISON WITH TESTS

Three two-span continuous prestressed beams tested by Bishara and Brar (1974) were analysed using the program SEGPCAN (acronym for Segmental Prestressed Concrete Analysis) developed based on the procedures already described. These are beams BC2, BC3 and BC4. As the beams were symmetrical about their centre supports, they were each modelled as a propped cantilever (see Fig 5). Each strand had an initial prestress of 61.38 kN. Effective prestress was assumed to be 80% of the initial prestress. The Young's Modulus of prestressing strand was assumed to be 194 000 MPa. The stress-strain relation of the strand was assumed to be elastic-plastic, with the yield stress equal to the 2% offset stress of 1710 MPa. The stress-strain relation for non-prestressed reinforcing bars was also assumed to be elastic plastic. The mean yield strength assumed was 401 MPa, and the Young's Modulus was assumed to be 200 000 MPa. The mean concrete strength f_{cm} for beams BC2, BC3 and BC4 were 42.2 MPa, 39.0 MPa and 39.0 MPa respectively. The initial modulus of concrete was assumed to be $5050\sqrt{f_{cm}}$. The shape of the concrete stress-strain relation for concrete in compression was assumed to be that proposed by Warner (1969) and for concrete in tension, including the effect of tension stiffening, was assumed to be that proposed by Kenyon and Warner (1993).

The load versus mid-span deflection plots for beams BC2, BC3 and BC4 are shown in Figs 6, 7 and 8 respectively. The results from the analysis agree reasonably well with the test results.

Two analyses were carried out on each beam, one including the effect of tension stiffening and the other without tension stiffening. For these beams, the results obtained from the analyses without tension stiffening agree well with test results. The results for the analysis which included the effect of tension stiffening over-stiffened the beams at low levels of loading.

5. HYPERSTATIC REACTIONS AND SECONDARY MOMENTS DUE TO PRESTRESS

In the linear analysis of prestressed concrete adopted for design purposes, it is normal to analyse for the effects of prestress alone, and to determine from this analysis the magnitude of the hyperstatic reactions and secondary moments which are produced by the deformations in the structure. Such hyperstatic reactions and secondary moments are only meaningful if it can be assumed that the structure behaves in a linear manner.

Prior to cracking of the concrete, any material non-linearity in structural behaviour is scarcely perceptible. However, in the present analysis the main attention is on non-linear behaviour in the post-cracking working load range and also on conditions at high overload. For this reason it has been assumed, even in the preliminary analysis of the application of prestress, that the behaviour is potentially non-linear. In this way, the analysis can proceed smoothly through each stage of loading. A legitimate alternative approach would be to assume linear behaviour prior to cracking and use the normal linear analysis for this initial stage.

The advantage of the linear analysis would be that any hyperstatic effects could be evaluated. The disadvantage is that an abrupt transition to non-linear analysis has to be introduced, with some slight mismatch of stresses and strains at the transition. The changeover would be when the first crack appears in the structure.

6. CONCLUDING REMARKS

The method of analysis described in this paper is more powerful than those methods previously used which rely on pre-generated moment-curvature relations for the individual elements. Generally it is similar to the finite element method developed by Scordelis, but it has the advantage that the analysis can be undertaken in either deformation or load control, so that good numerical stability is maintained when conditions at collapse are investigated.

It has the further advantage that it is a relatively simple modification of an existing method originally developed for reinforced concrete, which has been well tested and applied to various special problems. Special

analyses, for example allowing for non-proportional loading, can thus be carried out.

The comparisons with test data suggest that the method gives realistic results.

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APPENDIX A: STIFFNESS MATRIX FOR SEGMENTED ELEMENT

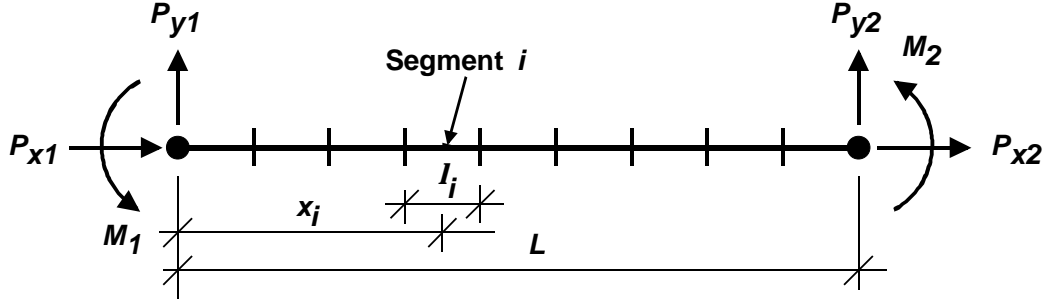


Figure A1: Segmented element showing element end forces

The force deflection relation for a segmented element (Figure A1) is :

$$\begin{Bmatrix} P_{x1} \\ P_{y1} \\ M_1 \\ P_{x2} \\ P_{y2} \\ M_2 \end{Bmatrix} = [K_e] \begin{Bmatrix} \Delta_{x1} \\ \Delta_{y1} \\ \theta_1 \\ \Delta_{x2} \\ \Delta_{y2} \\ \theta_2 \end{Bmatrix} \quad (\text{A1})$$

The element stiffness matrix $[K_e]$ is:

$$\begin{bmatrix} S_{11} & 0 & 0 & -S_{11} & 0 & 0 \\ & S_{22} & S_{23} & 0 & -S_{22} & LS_{22} - S_{23} \\ & & S_{33} & 0 & -S_{23} & LS_{23} - S_{33} \\ \text{SYM-} & & & S_{11} & 0 & 0 \\ \text{ME-} & & & & S_{22} & S_{23} - LS_{22} \\ \text{TRY} & & & & & L^2 S_{22} - 2LS_{23} + S_{33} \end{bmatrix} \quad (\text{A2})$$

where:

$$S_{11} = S(m) \quad (\text{A3})$$

$$S_{22} = \frac{C_1}{C_1 C_3 - C_2^2} \quad (\text{A4})$$

$$S_{23} = \frac{C_2}{C_1 C_3 - C_2^2} \quad (\text{A5})$$

$$S_{33} = \frac{C_3}{C_1 C_3 - C_2^2} \quad (\text{A6})$$

$$C_1 = \sum_{i=1}^{\text{no.of seg.}} \frac{1}{\left(\frac{EI_i}{l_i}\right)} \quad (\text{A7})$$

$$C_2 = \sum_{i=1}^{\text{no.of seg.}} \frac{x_i}{\left(\frac{EI_i}{l_i}\right)} \quad (\text{A8})$$

$$C_3 = \sum_{i=1}^{\text{no.of seg.}} \frac{x_i^2}{\left(\frac{EI_i}{l_i}\right)} \quad (\text{A9})$$

L is the length of the element. $S(m)$ is the axial stiffness for the element. It is set to a large value for elements subjected mainly to flexure. EI_i and l_i are the flexural stiffness and length respectively of the segment i , and x_i is the distance from the left end of the element to the centre of the segment. The derivation of the matrix is given by Wong, Yeo and Warner (1988).

APPENDIX B: FIXED END MOMENTS FOR A SEGMENTED ELEMENT

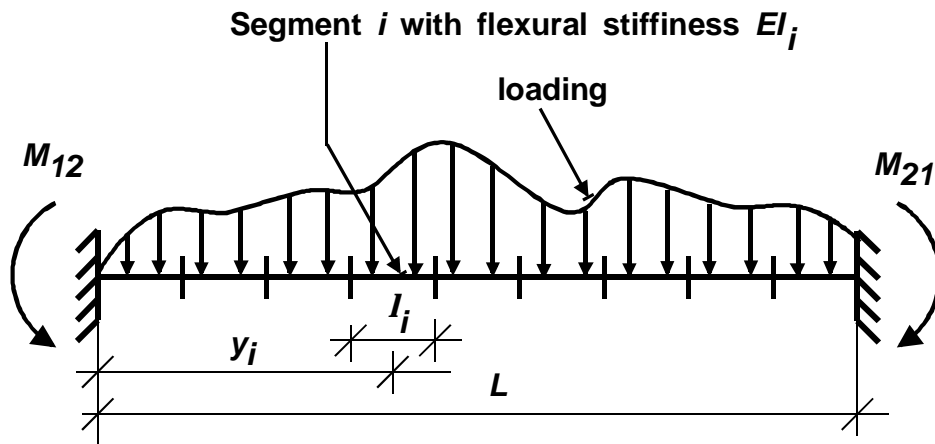


Figure B1: Segmented element showing fixed end moments

The fixed end moments M_{12} and M_{21} for the left and right ends of a segmented element (Figure B1) subjected to loading are:

$$M_{12} = \left[\frac{XT}{R} - \frac{S}{P} \right] \quad (\text{B1})$$

$$M_{21} = \left[\frac{(L-X)T}{R} - \frac{S}{P} \right] \quad (\text{B2})$$

X is the distance from the left end of the element to the centroid of the $1/EI$ diagram of the entire element:

$$X = \frac{\sum_{i=1}^{\text{no.of seg.}} \left(\frac{y_i}{EI_i} \right)}{\sum_{i=1}^{\text{no.of seg.}} \left(\frac{1}{EI_i} \right)} \quad (\text{B3})$$

y_i is the distance from the left end of the element to the centre of the segment i , and:

$$P = \sum_{i=1}^{\text{no.of seg.}} \frac{1}{\left(\frac{EI_i}{l_i} \right)} \quad (\text{B4})$$

$$R = \sum_{i=1}^{\text{no.of seg.}} \frac{x_i^2}{\left(\frac{EI_i}{l_i} \right)} \quad (\text{B5})$$

$$S = \sum_{i=1}^{\text{no.of seg.}} \frac{M_{ppi}}{\left(\frac{EI_i}{l_i} \right)} \quad (\text{B6})$$

$$T = \sum_{i=1}^{\text{no.of seg.}} \frac{M_{ppi} x_i}{\left(\frac{EI_i}{l_i} \right)} \quad (\text{B7})$$

EI_i and l_i are the flexural stiffness and length respectively of the segment i , and x_i is the distance from the centroid of the $1/EI$ diagram to the centre of the segment. M_{ppi} is the bending moment at the middle of segment i if the beam carrying the applied load were pin-supported at both ends.

APPENDIX C: FORCE FROM PRESTRESSING CABLE

This appendix shows how the prestressing forces are determined for a segmented element.

Case 1: At juncture between segments within the element:

For small angles, θ_1 and θ_2 :

$$\theta_1 \approx \sin\theta_1 = \frac{y_i - y_{i-1}}{l_{seg.}} \quad (C1)$$

$$\theta_2 \approx \sin\theta_2 = \frac{y_{i+1} - y_i}{l_{seg.}} \quad (C2)$$

The vertical point force acting upward directly above the junction between the two segments:

$$\begin{aligned} F_y &= F_{prest.}(\sin\theta_1 - \sin\theta_2) \\ &= F_{prest.}(\theta_1 - \theta_2) \\ &= \frac{F_{prest.}}{l_{seg.}}(y_i - y_{i-1} - y_{i+1} + y_i) \\ &= \frac{F_{prest.}}{l_{seg.}}(-y_{i-1} + 2y_i - y_{i+1}) \end{aligned} \quad (C3)$$

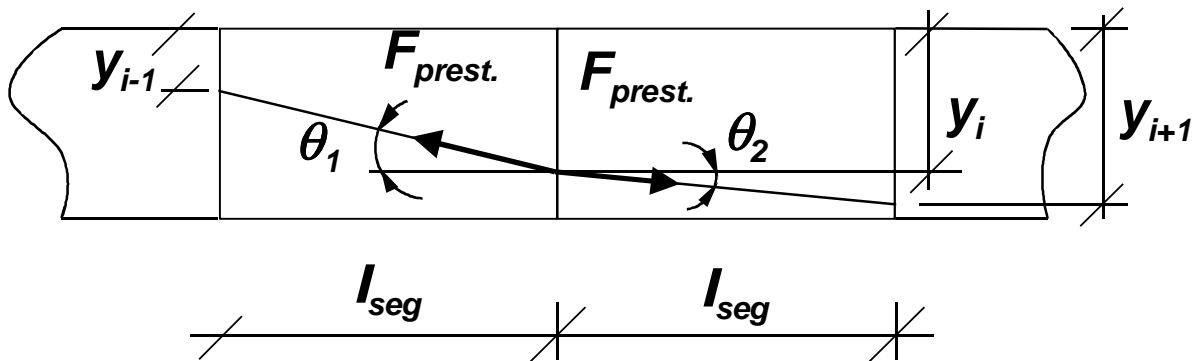


Figure C1: Prestressing effect at junction of two segments

Case 2: At the left end node of the segmented element:

For small angle,

$$\theta_2 \approx \sin\theta_2 = \frac{y_2 - y_1}{l_{seg.}} \quad (C4)$$

The vertical point force (positive upward) acting on the left node (from the removed tendon) is:

$$\begin{aligned} F_y &= -F_{prest.} \sin\theta_2 \\ &= -F_{prest.} \theta_2 \\ &= -F_{prest.} \left(\frac{y_2 - y_1}{l_{seg.}} \right) \end{aligned} \quad (C5)$$

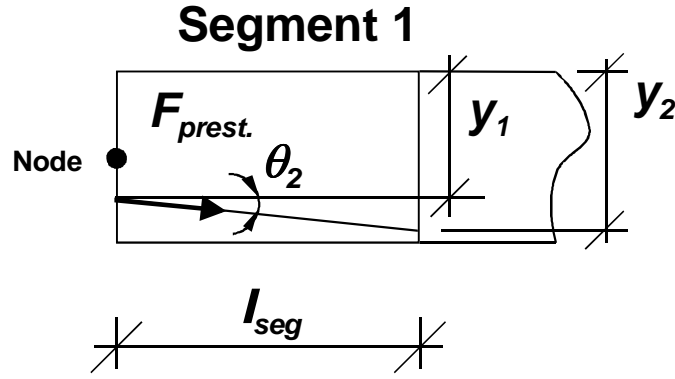


Figure C2: Prestressing effect at left node of segmented element

Case 3: At the right end node of the segmented element

For small angle,

$$\theta_1 \approx \sin\theta_1 = \frac{y_{nseg.+1} - y_{nseg.}}{l_{seg.}} \quad (C6)$$

The vertical point force (positive upward) acting on the right node (from the removed tendon) is :

$$\begin{aligned} F_y &= F_{prest.} \sin\theta_1 \\ &= F_{prest.} \theta_1 \\ &= F_{prest.} \left(\frac{y_{nseg.+1} - y_{nseg.}}{l_{seg.}} \right) \end{aligned} \quad (C7)$$

Last segment n_{seg}

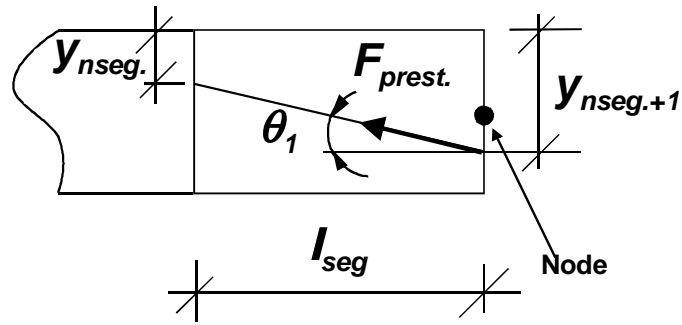
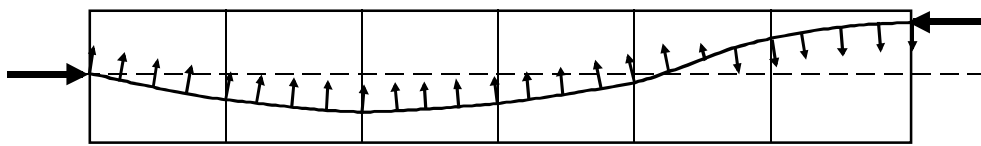
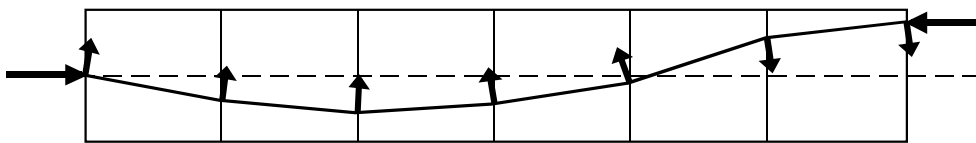


Figure C3: Prestressing effect at right node of segmented element

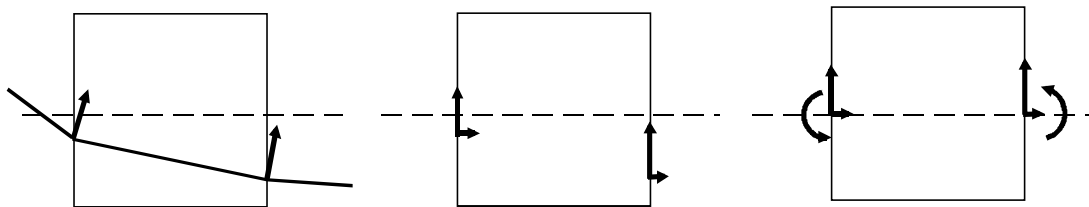


(a) Element with curved cable



(b) Piece-wise linear approximation with kinks at the junctures between segments

Figure 1: A typical segmented element



(a) Inclined local forces at junctures between segments (b) Vertical and horizontal components (c) resultants at junctures on centroidal axis

Figure 2: System of forces and moments acting at the junctures between segments

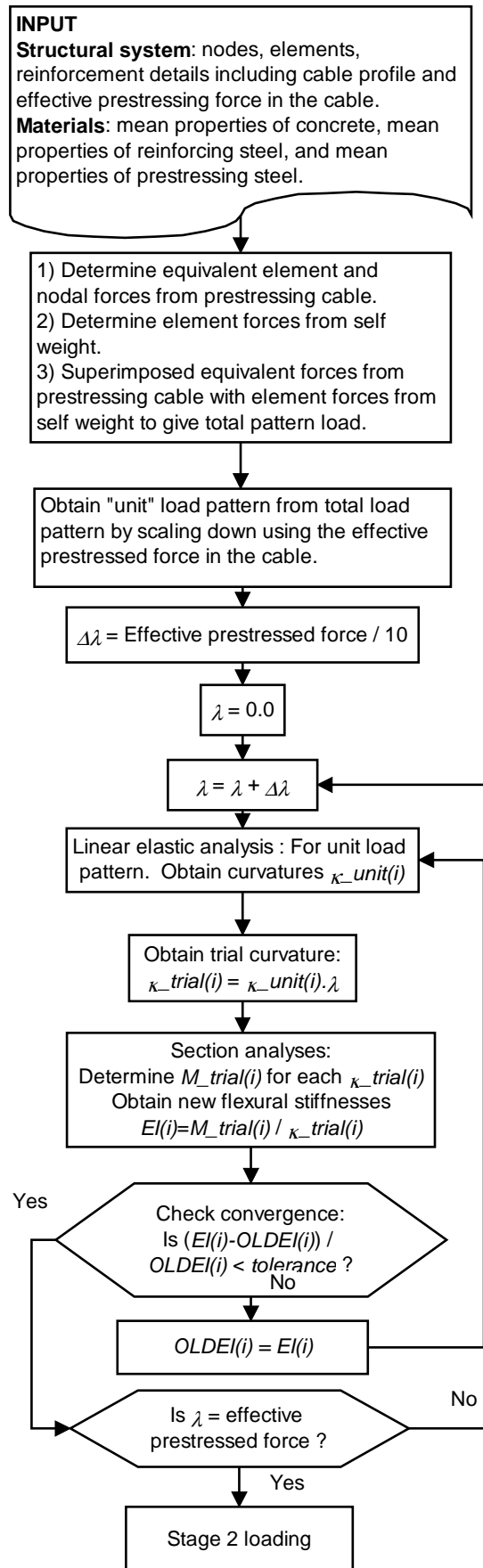


Figure 3: Stage 1 analysis

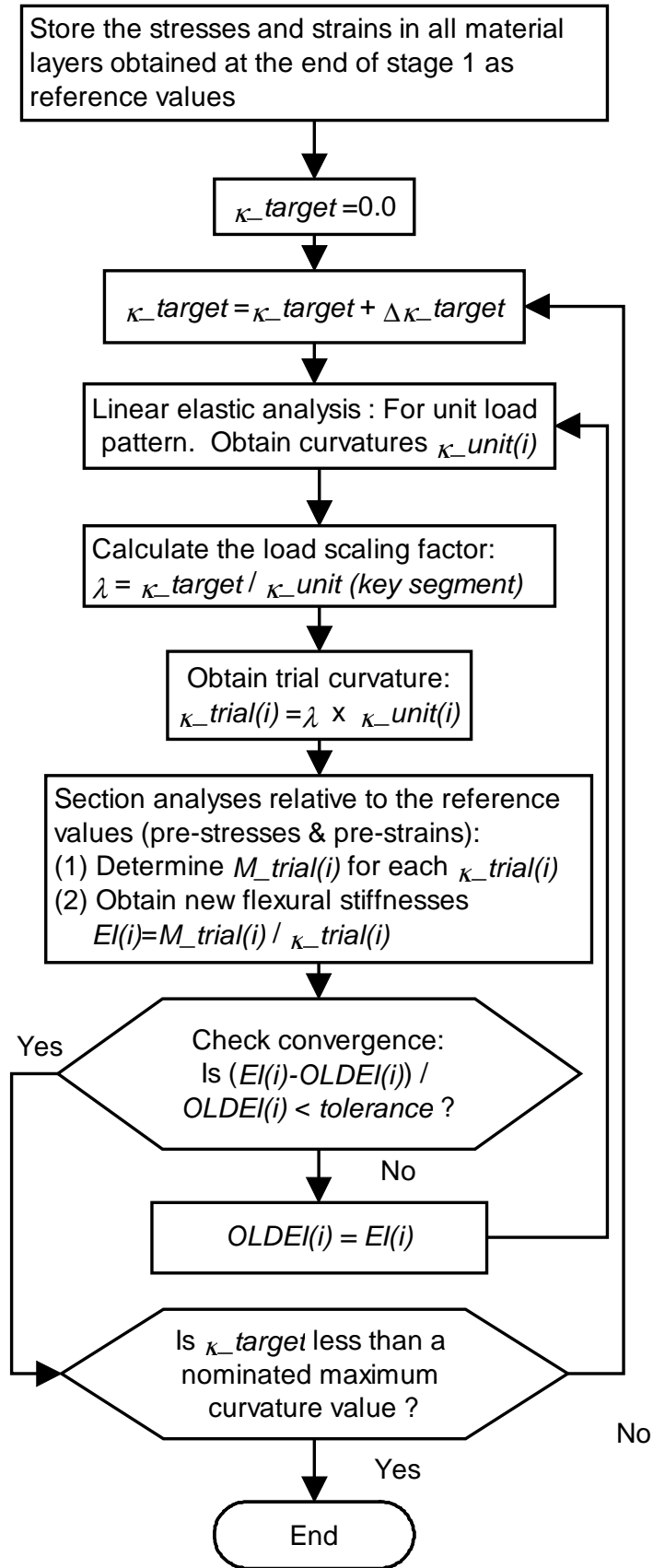
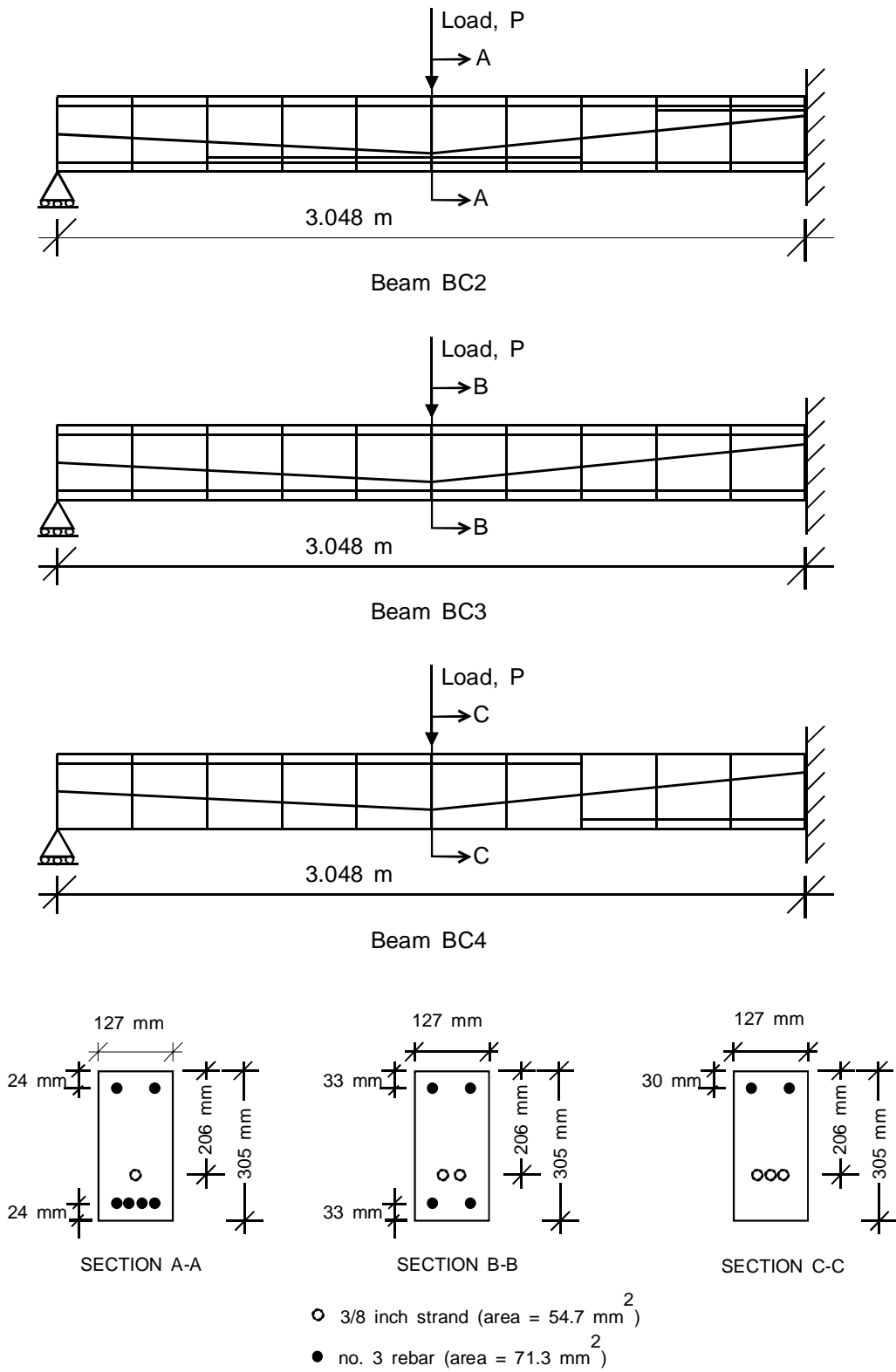


Figure 4: Stage 2 analysis



Note: Fixed support sections are the same as mid-span sections except inverted.

Figure 5: Details of beams tested by Bishara and Brar (1974)

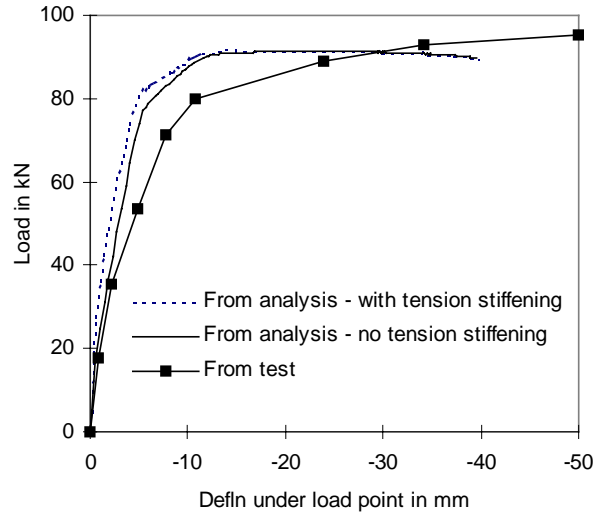


Figure 6: Load deflection plot for beam BC2

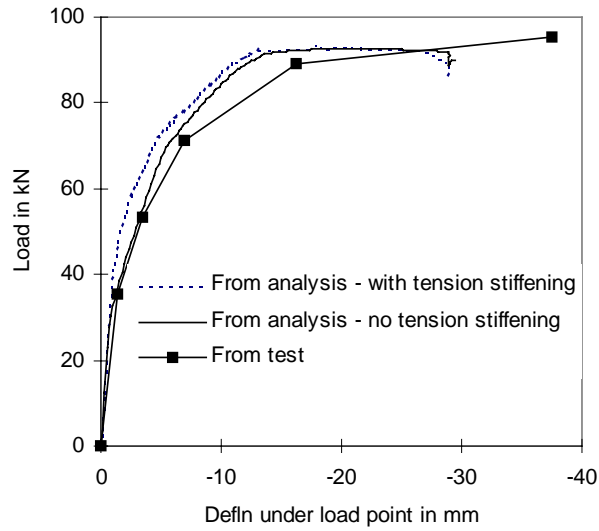


Figure 7: Load deflection plot for beam BC3

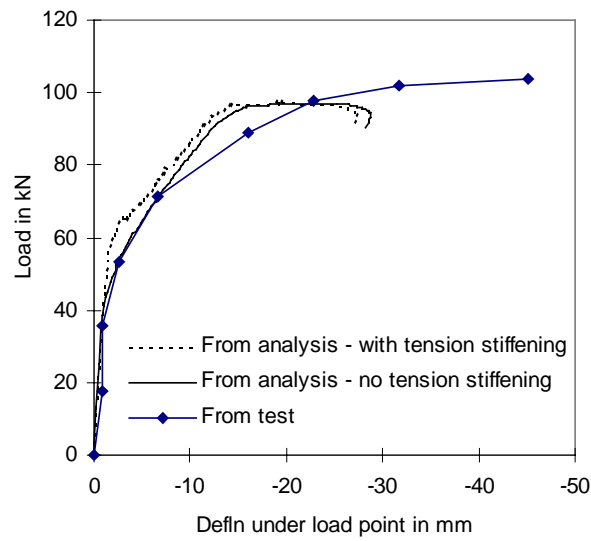


Figure 8: Load deflection plot for beam BC4