Non-linear Design of Reinforced Concrete Columns and Frames

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ABSTRACT: This report describes the evaluation of system safety coefficients for non-linear design of reinforced concrete columns and frames using a back-calibration method. The back-calibration method was described in a previous paper, and was used to evaluate system safety coefficients for continuous concrete beams. Values for system safety coefficients are proposed for ductile flexural systems, for systems in which collapse is due to concrete crushing, and for systems in which collapse is by loss of stability. The study of slender frames has shown up an inadequacy in the simplified column design procedure of AS 3600 when applied to slender frames.

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1. INTRODUCTION

In a previous paper (Wong and Warner, 1997a) back-calibration was proposed as a method of evaluating system safety coefficients for use in the non-linear, collapse load design of concrete structures. An extensive numerical study has also been undertaken to obtain appropriate values for the non-linear design of continuous reinforced concrete beams (Wong and Warner, 1998). For ductile flexural members it was found that the system safety coefficient value of

$$\phi_{system} = 0.68 \tag{1}$$

was appropriate. This value can thus be used in the collapse load design equation,

$$\phi_{system} w_{u.rig} \geq w^{*} \tag{2}$$

where w^* is the design ultimate load, and $w_{u.rig}$ is the collapse load, calculated using a rigorous non-linear analysis.

The advantages of moving on from the current section strength design methods to collapse load design have also been discussed elsewhere (Wong and Warner, 1997a).

In this report, the back-calibration method is used to evaluate ϕ_{system} for other structural systems, in particular columns and frames, in which collapse may be governed by compression failure of the concrete and overall system instability.

To apply the back-calibration method, a structure is first designed according to the present code requirements, in this case, AS 3600 (Standard Australia, 1994), to carry a prescribed design ultimate load, say w^*. A rigorous, non-linear collapse load analysis is then undertaken to obtain its collapse load, $w_{u.rig}$, and the safety coefficient is then evaluated as

$$\phi_{system} = \frac{w^{*}}{w_{u.rig}} \tag{3}$$

The assumption here is that the same design loads and load combination factors will be used to determine w^* in both the section strength design method and the collapse design load method, so that design using Eqn 2 gives a structure with about the same safety margin as would be obtained from the current design procedures.

2. NON-LINEAR VALUES OF SYSTEM SAFETY COEFFICIENT FOR ISOLATED BRACED COLUMNS

Back-calibration calculations were initially undertaken for 70 slender columns with pin supports and equal end eccentricities as shown in Fig 1. Their cross sections were all 400 mm x 400 mm. The total amount of reinforcement was 3200 mm^2, with a concrete cover to centroid of steel reinforcement of 50 mm. Slenderness ratio L_e/r of the columns ranges from 17 to 121, and three values of end eccentricities e/D of 0.0625, 0.75 and 4.0 were used. Here e is the load eccentricity and D is the overall depth of the section. Note that the minimum end eccentricity e/D of 0.0625 is comparable to the minimum e/D of 0.05 specified by AS 3600. The following values were used: a characteristic strength of concrete of 32.0 MPa and a mean value of 37.2 MPa, and a steel yield stress of 400 MPa and a mean value of 460 MPa. The steel reinforcement is two percent of the gross cross-sectional area.

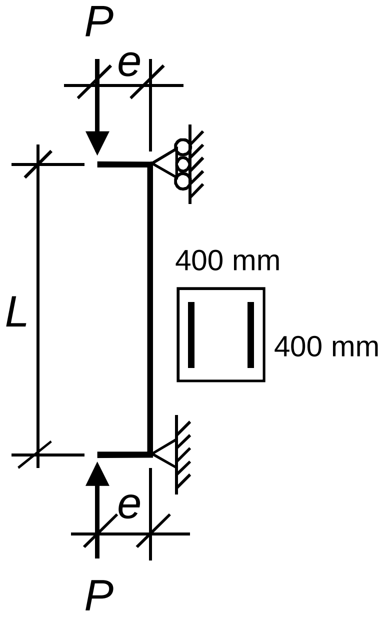

Figure 1: Braced column

As described in Section 1, the back-calibration procedure requires values of w^*, the design ultimate load, and $w_{u.rig}$, the accurately determined load capacity, for a wide range of structures. In the normal design situation, w^* is known and by trial and error design details are found such that for critical sections, $\phi M_u \geq M^*$. However, in the present investigation we fixed the design details, and then determined both w^* and $w_{u.rig}$ for each particular structural system. The calculation of $w_{u.rig}$ was carried out using the procedure described by Wong and Warner (1997b).

As the calculation of $w_{u.rig}$ does not include long-term effects such as creep and shrinkage, the creep factor β_d (defined by AS 3600 as the ratio of the dead load to the sum of the dead load plus live load) used in the calculation of the corresponding w^* was assumed to be zero.

The system safety coefficients for all the columns investigated are shown as a function of their slenderness ratios and end eccentricities in Fig. 2. The enormous scatter in results, with ϕ_{system} ranging from about 0.33 to 0.70, is not unexpected and can be explained by the peculiarities of the simplified methods used in AS 3600 for the analysis and design of columns.

Considering firstly the highest range of values, which occur for e/h varying from 4.0 down to, say, 0.75, we note that these all correspond to primary tension flexural failure in the mid-depth section of the column. The capacity reduction factor ϕ used in AS 3600 is close to 0.8 for these cases because ϕ for columns slides from 0.6 up towards 0.8 as the load capacity of the column falls below the load which produces 'balanced' failure.

On the other hand, at smaller values of e/h (e.g. below 0.5) and relatively small slenderness the values of ϕ_{system} cluster around 0.5. For these columns, failure is by primary compression and the capacity reduction factor ϕ from AS 3600 is 0.6. Furthermore, the effect of slenderness on strength is minimal at L_{eff}/r values of around 20. Thus if we reduce the value of ϕ_{system} for beams (0.68) by the ratio 0.6/0.8, we obtain a value of about 0.5.

On the other hand, with progressively increasing slenderness the ϕ_{system} values diverge, usually upwards, but for the smallest eccentricities ($e/h \leq 0.125$) the divergence is severely downwards. This reflects the very conservative nature of the AS 3600 moment magnification factor method at very small eccentricities and high slenderness.

The increase in calculated values of ϕ_{system} with increasing slenderness, as shown in Fig 2 for *e/h* values of 0.5 and 0.25, requires explanation. It appears that at low slenderness the failure mode is in primary compression; however, with increasing slenderness there is sufficient lateral bending and outward movement of the central region of the column to lead to primary flexure failure and hence to the use of increasing ϕ values greater than 0.6. This view of column behaviour is of course highly simplified and while convenient for design, is inaccurate. For example, it can be shown that stability failure can precede local section failure in a real column of finite length (Warner et al, 1989).

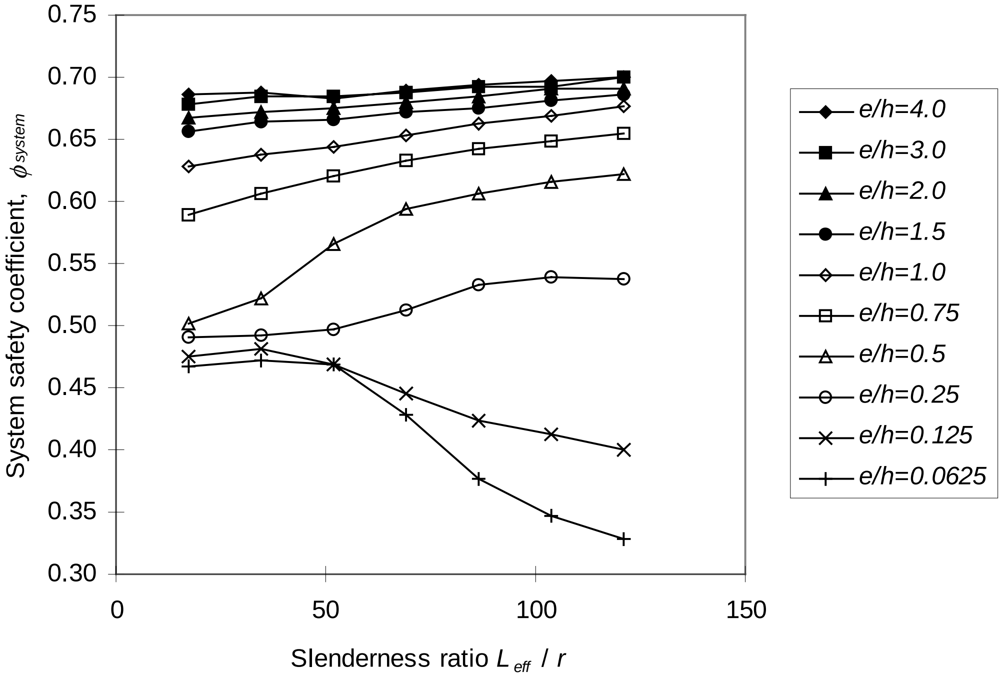

Figure 2: System safety coefficients for braced columns

The results shown in Fig 2 allow some preliminary suggestions to be made for values of ϕ_{system}. Working from the basic ϕ_{system} value of 0.68 for ductile continuous beams, a smaller value, say 0.5 seems appropriate, for cases where collapse is due to compression type failure, and a further reduction, perhaps to 0.4, in cases of high slenderness, where stability failure dominates. It is emphasised that these are only indicative figures which will have to be confirmed or modified by additional modelling.

3. EVALUATION OF SYSTEM SAFETY COEFFICIENTS OF PORTAL FRAMES BY BACK-CALIBRATION

In this initial study of safety coefficients for structural frames, attention has been concentrated on simple portal frames. It should be noted that frames with ductile strong columns and weak beams with uniformly distributed loads acting along the beams fail when a plastic collapse mechanism occurs in the beam. For such cases the value of $\phi_{system} = 0.68$ can be used for design (Wong and Warner, 1997a). Of interest here are frames which may fail by bending in both the beams and columns, and overall instability. The portal frames shown in Fig 3 were therefore used in the study, with a horizontal load H at floor level and point loads P on the columns, with the ratio H/P varying from 0.1 up to 1.5. Both fixed bases (as shown) and pinned bases were considered, with three frame heights of 4m, 6m, and 8m for the fixed-base frames; and two frames heights of 4m and 6m for the pinned-base frames. The slenderness ratios L_{eff}/r for the columns for the fixed-base frames, calculated in accordance with AS 3600, were 46, 66 and 85; and those for the pinned-base frames, 67 and 95. The total amount of reinforcement was 3200 mm^2 in the cross-section of the columns, with a concrete cover to centroid of steel reinforcement of 50 mm. Proportional loading was assumed, with P and H applied simultaneously, although a recent study suggests that the results would not be significantly different with a non-proportional P-H load sequence (Wong and Warner, 1997c).

For each frame the load capacity $P_{u.rig}$ was calculated by an accurate non-linear analysis (Wong and Warner, 1997b), and the design ultimate load P^* determined in accordance with the simplified approach of AS 3600. The system safety coefficient was then determined:

$$\phi_{system} = \frac{P^*}{P_{u.rig}} \tag{4}$$

The system safety coefficient values are shown in Tables I and II for the fixed base and pinned base frames, respectively, together with the moment magnification factor δ_s used in the design of the columns in the frames. The values of ϕ_{system} are also shown graphically in Fig 4 as a function of the load ratio H/P.

In the case of the fixed base frames, the larger end moments in the columns are in the lower ends, next to the fixed-bases. Hinges formed first in the lower ends of the column, followed by the formation of hinges next to the ends of the beams. The values of ϕ_{system} close to 0.68 obtained for the frames is due to the formation of the second set of hinges in the beams at the beam column joints rather than in the columns, thus limiting the amount of load the frames can carry. The formation of the second set of hinges in the beams rather than in the columns reflects the strong-column weak-beam design which resulted from the simplified approach of AS 3600. The system safety coefficients are thus closer to those associated with ductile beam failure, rather than the more conservative values associated with column instability failure.

There are noticeable increases in safety margin in the three frames with column sections having the design strength governed by primary compression failure. The extra conservativeness is due to the use of a section strength design reduction factor ϕ of 0.6 for columns which failed by primary compression failure when compared with a value of between 0.6 to 0.8 for those which failed by primary tension failure.

The moment magnification factor δ_s is also reasonably close to unity for H/P values at 0.5 and greater in the case of the pinned-base frames, although for some reason the values of ϕ_{system} are consistently higher than the expected value of 0.68 for H/P values at 0.5 and greater. The larger end moments in the columns are next to the connecting beams.

The values of δ_s increase markedly as the value of H/P decreases below 0.5. Quite unexpectedly the ϕ_{system} value increases. The frame which satisfies the design requirement of AS 3600 and has the least safety margin is the 4m tall frame with $H/P = 0.1$; it has a ϕ_{system} value of 0.89. This suggests that the current design methods give relatively less conservative results for pinned-base portal frames with large moment magnifiers δ_s in the columns.

A previous study by Pagay et al(1970) shows that the strength of beams has a pronounced effect on the strength of concrete frames, even though some present design standards fail to take proper account of this. For example, AS 3600 does not specify any requirement for increasing the reinforcement in adjoining beams, to take account of the additional moments transferred from the ends of the columns. Reinforcement in beams is incorrectly obtained using the action effects from a linear elastic analysis, assuming gross section properties for members.

A reassessment of the system safety coefficients was made for the pinned-based portal frame, but with the beams designed to carry the increased bending moment obtained from the first-order elastic analysis, magnified by the sway moment magnifier δ_s. The system safety coefficients obtained are given in Table III and illustrated in Fig 5. All now lie close to the value of 0.68. This confirms that the reduced conservativeness was caused by the failure of AS 3600 to require an increase in the reinforcement of the beam to complement the moment magnification effect acting on slender columns. As observed in a previous preliminary study (Wong and Warner, 1997a) on frames, the present study confirms that the conservative moment magnifier approach of AS 3600 for the design of columns always results in beam failure in the frames.

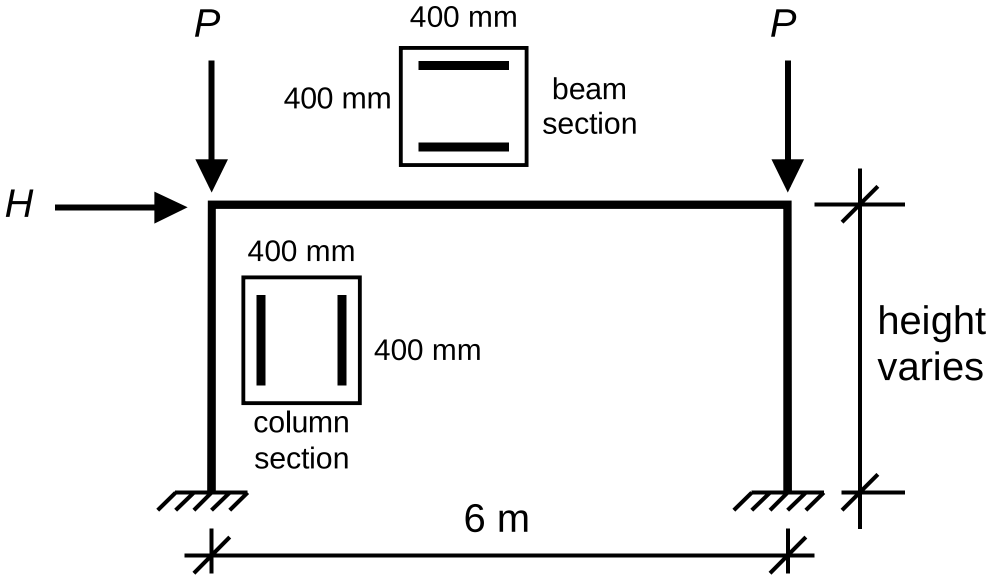

Figure 3: Fixed base portal frame

Table I

Moment magnifiers and system safety coefficients for fixed-base portal frames

	4m tall portal		6m tall portal		8m tall portal	
H/P	δ_s	ϕ_{system}	δ_s	ϕ_{system}	δ_s	ϕ_{system}
0.05	1.56*	0.51$^{\#}$	1.98*	0.60$^{\#}$	2.37*	0.68
0.10	1.34	0.53$^{\#}$	1.54*	0.63	1.63*	0.69
0.20	1.19	0.60	1.24	0.64	1.28	0.67
0.30	1.11	0.61	1.14	0.63	1.17	0.67
0.40	1.08	0.62	1.10	0.63	1.13	0.67
0.50	1.06	0.63	1.08	0.61	1.10	0.66
0.60	1.05	0.63	1.06	0.62	1.08	0.66
0.80	1.04	0.63	1.05	0.63	1.06	0.64
1.00	1.03	0.61	1.04	0.63	1.05	0.66
1.50	1.02	0.62	1.02	0.64	1.03	0.66

note: * exceeds maximum value of 1.50 allowed in AS 3600

$^{\#}$ primary compression failure in critical column section in simplified design method of AS 3600

Table II

Moment magnifiers and system safety coefficients for pinned-base portal frames

	4m tall portal		6m tall portal	
H/P	δ_s	ϕ_{system}	δ_s	ϕ_{system}
0.10	1.47	0.89	1.56*	0.93
0.20	1.20	0.78	1.25	0.81
0.30	1.12	0.76	1.16	0.76
0.40	1.09	0.73	1.11	0.73
0.50	1.07	0.69	1.09	0.72
0.60	1.06	0.70	1.07	0.71
0.80	1.04	0.68	1.05	0.69
1.00	1.03	0.68	1.05	0.69
1.50	1.02	0.71	1.05	0.69

note: * exceeds maximum value of 1.50 allowed in AS 3600

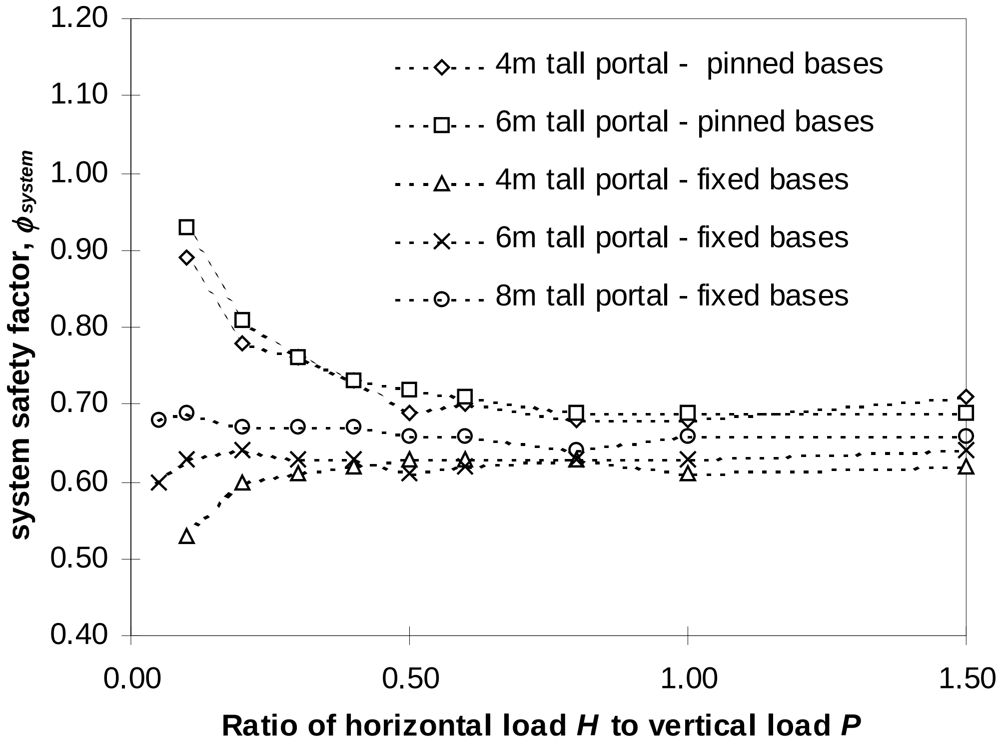

Figure 4: System safety coefficients for portal frames

Table III
Moment magnifiers and system safety coefficients for pinned-base portal frames with increased beam reinforcement

	4m tall portal		6m tall portal	
H/P	δ_s	ϕ_{system}	δ_s	ϕ_{system}
0.10	1.47	0.66	1.56*	0.68
0.20	1.20	0.67	1.25	0.68
0.30	1.12	0.68	1.16	0.67
0.40	1.09	0.68	1.11	0.67
0.50	1.07	0.66	1.09	0.68
0.60	1.06	0.67	1.07	0.68
0.80	1.04	0.64	1.05	0.67
1.00	1.03	0.67	1.05	0.70
1.50	1.02	0.70	1.05	0.68

note: * exceeds maximum value of 1.50 allowed in AS 3600

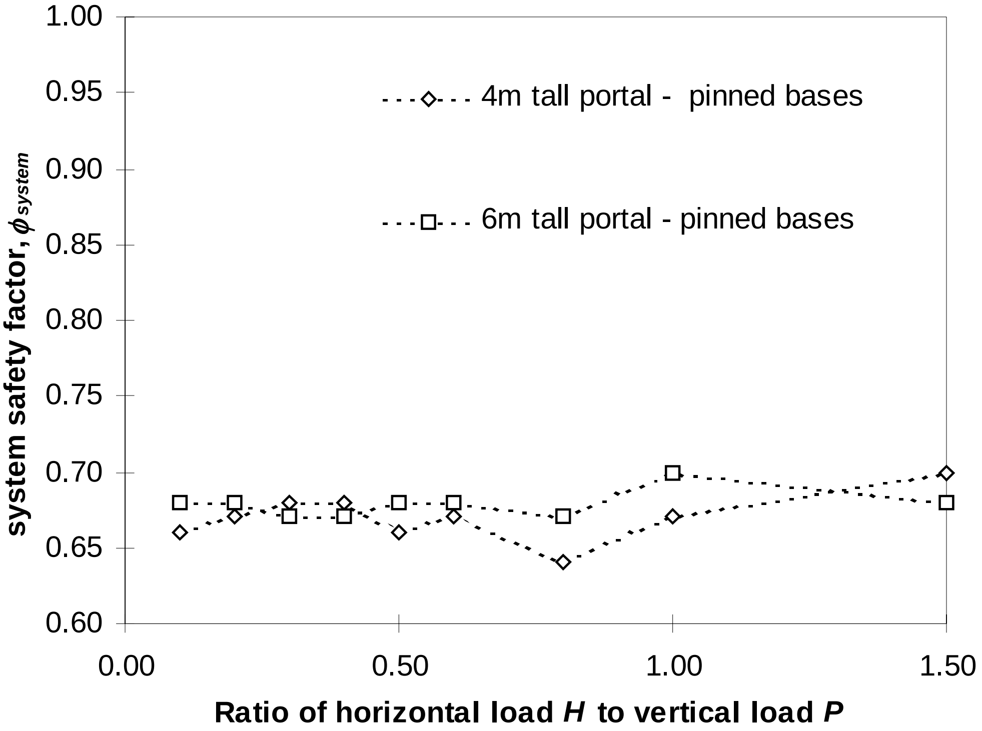

Figure 5: System safety coefficients for pinned-base portal frames with increased reinforcement in beams

4. CONCLUSIONS

The system safety coefficients obtained for the portal frames analysed generally gave values of approximately 0.68. Several of these portal frames gave larger, and therefore less conservative, system safety coefficients. However, this was found to be caused by a deficiency in AS 3600 in not magnifying moments in beams connected to slender columns. When account is taken of the increased moment in such beams, the system safety coefficients of these frames were found to be close to 0.68.

In the extensive study carried out using a back-calibration procedure for pin-ended columns the system safety coefficients for columns designed using the simplified approach of AS 3600 range from 0.33 to 0.70. On investigation, it was shown that the AS 3600 simplified design method can give highly conservative results, and this explains the exceptionally low values sometimes obtained for ϕ_{system}.

The present studies on columns and frames need to be extended. Further work will have to be carried out before reliable system safety coefficients can be proposed for the entire range of structures. Nevertheless, appropriate values of ϕ_{system} for structures with normal strength concrete and f_{sy} = 400 MPa reinforcing steel are likely to be:

- For ductile flexural systems and flexural collapse mechanisms: $\phi_{system} = 0.68$
- For isolated columns with small load eccentricities and large slenderness ratios: $\phi_{system} = 0.40$
- For frames which fail by loss of stability: $\phi_{system} = 0.40$
- For systems in which collapse is due to crushing of concrete, such as short axially loaded column: $\phi_{system} = 0.50$

In structural systems where the mode of collapse is not due to the formation of a flexural collapse mechanism, and the collapse cannot be attributed to the crushing of concrete, the more conservative value ϕ_{system} of 0.40 for instability failure should be used.

5. ACKNOWLEDGMENT

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